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Essays On Corporate Investment Dynamics

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Abstract
This dissertation contains two paper. The first, "Volatility and Venture Capital,'" demonstrates that the performance of venture capital (VC) investments load positively on shocks to aggregate return volatility. I document this novel source of risk at the asset-class, fund, and portfolio-company levels. The positive relation between VC performance and volatility is driven by the option-like structure of VC investments, especially by VCs' contractual option to reinvest. At the asset-class level, shocks to aggregate volatility explain a substantial fraction of VC returns. At the fund level, consistent with the reinvestment channel, this exposure is concentrated in years two through four of fund life and in early-stage VC funds, which have more embedded reinvestment options. For VC-backed portfolio companies, volatility shocks correlate with faster and more frequent reinvestment. The level of volatility at the time of investment has no relation with future performance, consistent with competitive markets. Overall, my results imply that the option-like features of VC investments are first-order determinants of risk in VC.

The second paper, "Intangible Capital and the Investment-q Relation," shows that the neoclassical theory of investment, which has mainly been tested with physical investment, also helps explain intangible investment. At the firm level, Tobin's q explains physical and intangible investment roughly equally well, and it explains total investment even better. Compared with physical capital, intangible capital adjusts more slowly to changes in investment opportunities. The classic q theory performs better in firms and years with more intangible capital: Total and even physical investment are better explained by Tobin's q and are less sensitive to cash flow. At the macro level, Tobin's q explains intangible investment many times better than physical investment. We propose a simple, new Tobin's q proxy that accounts for intangible capital, and we show that it is a superior proxy for both physical and intangible investment opportunities.

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ESSAYS ON CORPORATE INVESTMENT DYNAMICS

Dedicated to Rose and Kurt

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This dissertation contains two paper. The first, "Volatility and Venture Capital," demonstrates that the performance of venture capital (VC) investments load positively on shocks to aggregate return volatility. I document this novel source of risk at the asset-class, fund, and portfolio-company levels. The positive relation between VC performance and volatility is driven by the option-like structure of VC investments, especially by VCs' contractual option to reinvest. At the asset-class level, shocks to aggregate volatility explain a substantial fraction of VC returns. At the fund level, consistent with the reinvestment channel, this exposure is concentrated in years two through four of fund life and in early-stage VC funds, which have more embedded reinvestment options. For VC-backed portfolio companies, volatility shocks correlate with faster and more frequent reinvestment. The level of volatility at the time of investment has no relation with future performance, consistent with competitive markets. Overall, my results imply that the option-like features of VC investments are first-order determinants of risk in VC.

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1.1. Introduction

This paper identifies a novel source of risk that helps to explain the observed time-series patterns in venture capital (VC) investment performance: exposure to shocks to the level of aggregate idiosyncratic return volatility. While idiosyncratic shocks are by definition mean-zero return shocks, the level of idiosyncratic volatility drives VC returns through the value of the real options embedded in VC contracts. When volatility increases, these real options become more valuable, improving VCs' performance. This paper examines the extent to which this exposure can explain observed patterns in VC investment performance and dynamics and identifies the primary channel driving this empirical relationship.

Two common contractual channels of VC investments generate option-like characteristics. The first is the liquidation preference given to investors, which entitles them to recuperate at least their initial investment before other investors participate in proceeds from any potential firm exit. These liquidation preferences imply a nonlinear payoff structure similar to those in equity options. The second channel is the (real) reinvestment option embedded in the contracts that VCs write with their portfolio companies, namely a right of first refusal for participation in future financing rounds.[1]

There are a number of other reasons why idiosyncratic volatility might be especially important to the VC sector. First, VC returns are known to be highly skewed. Metrick and Yasuda (2011), among others, show that a very large share of the total returns in VC come from a small fraction of their investments. Additionally, the compensation structure of VC general partners (GPs) themselves (eg. two and twenty) encourages the construction of portfolios that have a high total return variance, and thus a high idiosyncratic volatility. Moreover, VCs traditionally invest in the types of firms (small, high-tech) whose idiosyncratic volatility is high and loads heavily on aggregate changes in idiosyncratic volatility.

[1] Bergemann and Hege (1998, 2005) demonstrate that these contractual features emerge from the contracting environment with learning and moral hazard.

While each of these characteristics of VC investments suggest that idiosyncratic volatility may be particularly large, none of them suggest that changes in the level of idiosyncratic volatility should drive returns.

I investigate the empirical relation between VC investments and innovations in aggregate idiosyncratic volatility at three levels of aggregation. First, at the asset class level, I measure the exposure of VC benchmark portfolios to changes is aggregate idiosyncratic volatility. Second, I use detailed data on investor cash flows to investigate the time heterogeneous exposure of individual VC funds to innovations in asset volatility during those funds' lives. The third level of aggregation is at the level of individual investments by VCs into their portfolio companies, where I investigate the extent to which idiosyncratic volatility is related to the process by which VCs invest.

Rather than directly constructing a measure of VC portfolio company idiosyncratic volatility, I use a measure of the idiosyncratic volatility of publicly traded equities from Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) as a proxy for the idiosyncratic volatility of VC-backed firms. I use a public market proxy for two reasons. First, return data on publicly traded equity is of better quality and available at a much higher frequency, allowing for high-frequency estimates of idiosyncratic volatility shocks. The second reason is that using a measure from public markets minimizes potential endogeneity concerns.[2]

There are a number of reasons to believe that the idiosyncratic volatility of VC-backed firms is strongly related to that from public markets. First, Herskovic et. al. (2016) show that publicly traded firms' idiosyncratic volatility obeys a strong factor structure, i.e. that the level of idiosyncratic volatility is highly correlated across firm sizes and industries. Additionally, they find that the idiosyncratic volatility of small firms along with high-tech and health-related firms, the types of firms typically financed by VC funds, are more sensitive to this common factor. Finally, I validate the measure by comparing low frequency estimates

[2]One potential source of endogeneity is the fact that the amount of experimentation done by VCs, and therefore the idiosyncratic volatility of VC-backed firms, may be endogenously determined by the amount of capital available to VCs for investment as argued by Nanda and Rhodes-Kropf (2013, 2016).

of VC-backed firm idiosyncratic volatility to similarly timed low frequency measures from publicly traded firms and show that these two series move together strongly.

I begin the main empirical analysis by establishing that commonly used VC benchmark return indexes load positively on idiosyncratic volatility shocks. In particular, factor regressions for these VC indexes have significantly higher R^2 when including idiosyncratic volatility shocks. In other words, idiosyncratic volatility shocks explain a large fraction of the variance of VC industry returns after accounting for the fraction explained by market returns. I incorporate lagged regressors in these factor regressions to account for asynchronous prices of privately traded firms as discussed in Dimson (1979), and find that a one quarterly standard deviation innovation to the level of idiosyncratic volatility corresponds to an approximately 8% return in the VC index. This effect is even larger for a benchmark index of early-focused VC performance.

If the price of volatility risk is non-zero, this exposure has implications for the risk-adjusted performance of VC investments. There is a large body of theoretical and empirical work[3] that suggests the price of volatility exposure is negative, i.e. that investors are willing to pay for exposure to this risk. In this case, the positive exposure to this risk documented in this paper implies that the risk-adjusted performance of VC investments is higher than previously believed. I test this hypothesis by measuring the exposure of VC benchmark return indexes to a tradeable portfolio of equity options constructed to proxy for idiosyncratic volatility risk, and find that accounting for this tradeable factor increases the risk-adjusted performance of venture capital investments by as much as 6% per year.

Next, in order to relate these shocks to investor cash flows directly and investigate the potential channels driving this empirical relationship, I examine the exposure of individual venture capital funds to changes in aggregate idiosyncratic volatility. I use the realized cash flows between investors and the VC funds in which they invest from Burgiss. This data is

[3]See Mankiw (1986), Constantinides and Duffie (1996) and Herskovic, Kelly, Lustig and Van Nieuwerburgh (2016).

sourced from a diverse array of limited partners (fund investors) for whom Burgiss provides investment decision support tools and includes a complete transaction and valuation history between the LPs and fund investments.

The fund performance measure I use is the public market equivalent (PME) of Kaplan and Schoar (2005), which accounts for the opportunity cost of capital.[4] I find a significant relation between fund PME and idiosyncratic volatility shocks over the life of the fund and, in particular, those shocks that occur in years two and three of the fund's life. This finding is consistent with the reinvestment option channel, since by year two most initial investments will have been made, and by year 5 VC fund managers are looking to exit their positions. I also estimate the strength of this exposure in funds with different investment focus. The relation is much stronger for those funds which focus on early-stage investments, where reinvestment options inherently plays a larger role, than in late-stage investment focused funds. In a placebo analysis, I show that buyout funds, which buy target companies whole and therefore have no reinvestment options, exhibit no exposure to volatility shocks. This suggests that exposure to aggregate idiosyncratic volatility is not a feature of private equity investments more broadly.

At the level of VC investments into portfolio companies, I use cash-on-cash multiples and annualized returns as the measures of performance, controlling throughout for public equity prices (Tobin's q) and returns. I find that investments made when idiosyncratic volatility is high do not have higher average returns than those made at other times, consistent with the level of idiosyncratic volatility being priced into individual VC financing deals, as would be expected. What does drive investment-level returns are shocks to the level of idiosyncratic volatility after the initial investment. This relationship is again much stronger for early investment rounds, consistent with the reinvestment channel. I also find that reinvestments happen faster and are more likely in times of rising idiosyncratic volatility,

[4]Specifically, the PME provides a valid economic performance measure when the LP has log-utility preferences and the return on the LP's total wealth equals the market portfolio (Sorensen and Jagannathan, 2015). I also examine the effect of loosening these restrictions, as in, for example, Korteweg and Nagel (2016).

suggesting that these innovations have a direct relationship with the amount of capital available to entrepreneurs.

Taken together, these results imply a strong effect of aggregate idiosyncratic volatility shocks on investment returns and dynamics in VC investments. This effect can help us to understand the time-series of VC investment dynamics and rationalize the large differences in risk-adjusted returns observed at different times in the existing literature. Moreover, the results imply that the real options embedded in VC investments are a first-order determinant of risk in VC.

This paper relates to the literature surrounding the impact of contractual and informational frictions on VC investments. In particular, Ewens, Jones and Rhodes-Kropf (2013) demonstrate how the principal-agent problem between VCs and their investors cause private equity fund returns to depend on diversifiable risk in the cross section of funds. This effect is distinct from the time-series relationship investigated in this paper. In addition, their cross-sectional results hold for both VC and buyout funds, while the results in this paper hold only for VC funds. This result makes sense because the reinvestment options which are the focus of this paper are only present in VC investments. Cornelli and Yosha (2003) show that staged financing helps to mitigate the problem of manipulation for purposes of window-dressing. Fluck, Garrison, and Myers (2007) highlight the role of staged financing as a real option and show that it alleviates the effort provision problem.

There is also a substantial literature on the risk and return characteristics of VC investments: Cochrane (2005), Kaplan and Schoar (2005), Hall and Woodward (2007), Korteweg and Sorensen (2010), and Korteweg and Nagel (2016). A large literature on the implications of idiosyncratic risk for entrepreneurs and managers is summarized by Heaton and Lucas (2004) and Hall and Woodward (2010). This paper differs from the previous literature by explicitly accounting for the role of idiosyncratic volatility in return dynamics.

This paper also relates to a large empirical literature on the role of idiosyncratic return volatility. Campbell, Lettau, Malkiel, and Xu (2001) examine secular variation in average idiosyncratic return volatility. Wei and Zhang (2006) study aggregate time-series variation in fundamental volatility. Engle and Figlewski (2015) document a common factor in option-implied volatilities. Jurado, Ludvigson, and Ng (2015) study measures of uncertainty from aggregate and firm-level data and relates them to macroeconomic activity. Ang, Hodrick, Xing, and Zhang (2006) show that stocks with high idiosyncratic volatility earn abnormally low average returns. This paper adds to this literature by identifying VC as a sector that is particularly exposed to idiosyncratic volatility risk.

The rest of the paper proceeds as follows: Section 1.2 presents two potential channels of volatility exposure. Section 1.3 introduces the VC data and the idiosyncratic volatility measure and compares public and private market idiosyncratic volatility. Section 1.4.1 presents empirical results using VC-industry return indexes. Section 1.4.2 presents empirical results using investor cash flows while section 1.4.3 presents empirical results using investment-level data. Section 1.5 concludes.

1.2. Sources of Volatility Exposure

This section briefly describes two potential channels through with VC investments may be exposed to shocks to the volatility of their underlying assets. The first channel is through the liquidation preference given to investors through the convertible preferred equity structure common to VC investments. This contractual feature of VC contracts induces both concavity and convexity in the VC payoff, which can lead to volatility exposure. The second channel is the (real) reinvestment option embedded in the contracts that VCs write with their portfolio companies. These contracts frequently include a contractual right of participation in future investment rounds which can again lead to a volatility exposure of the vale of the security. Both liquidation preferences and staged investment emerge from the contracting environment with information asymmetry (Bergemann and Hege (1998, 2005)), an inherent friction in the financing of small, private firms. A third potential explanation of the idiosyncratic volatility exposure of VC is that the underlying assets, the portfolio companies themselves, are positively exposed to volatility shocks.

1.2.1. Liquidation Preferences

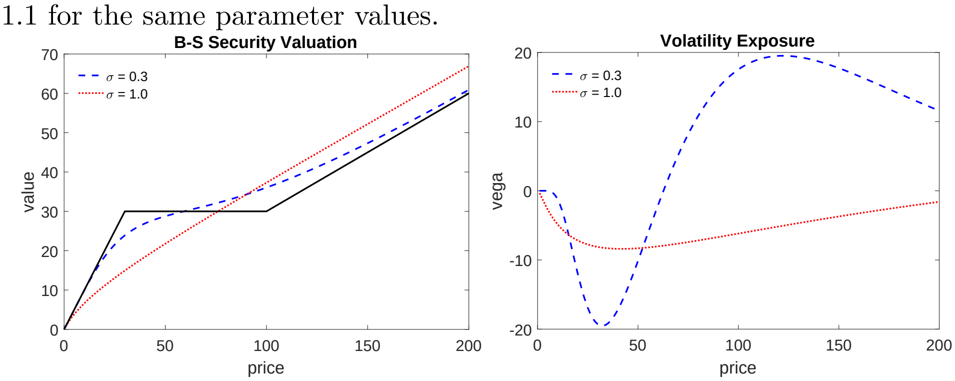
Kaplan and Stromberg (2003) report that of 213 rounds of financing, all but one contained some form of liquidation preference. A standard specification of the liquidation preference takes the following form in the National Venture Capital Association's (NVCA) 2013 model term sheet:

> In the event of any liquidation, dissolution or winding up of the Company, the proceeds shall be paid as follows: First pay [x] times the Original Purchase Price ... on each share of Series A Preferred. Thereafter, the Series A Preferred participates with the Common Stock pro rata on an as-converted basis.

Typically, the liquidation preference amounted to the lesser of the liquidation value of the firm and the VC's original investment ($x = 1$) and may include an unpaid cumulative dividend (45% of cases) that raises this amount over time.

Figure 1: Convertible Preferred Equity

Panel (a) shows the intrinsic and total value of convertible preferred equity stake. The solid line is that payoff that the holder of the the convertible preferred equity receives at a given exit value. The dashed (blue) and dotted (red) lines are the value of holding the convertible preferred equity as calculated using the Black-Scholes model. The dashed blue and dotted red lines assumes an annualized volatility of 0.3 and 1.0, respectively. For both lines the interest rate and dividend yields are zero and the time to expiration is three years. Panel (b) shows the vega, or volatility exposure, of the convertible preferred equity as calculated in Eq. 1.1 for the same parameter values.



(a) (b)

Panel (a) of Figure 1 shows a payoff diagram for a typical convertible preferred equity stake.[5] The dashed line represents the payoff to a participating convertible preferred equity investor holding a certain ownership share in the firm (in this case 30%). The dashed (blue) line represents the value before the final payoff is realized, as calculated using the Black-Scholes model assuming an annualized volatility of 30% while the dotted (red) line assumes an annualized volatility of 100%, which is close to the average idiosyncratic volatility of VC-backed firms (see Section 1.3.5).[6] The distance between the dotted and solid lines represents the extrinsic value of the option and is directly related to the volatility of the underlying asset. The volatility exposure, or *vega*, of a long call option is positive and easily calculated as:

$$\frac{\partial C}{\partial \sigma} = S\sqrt{\frac{T}{2\pi}}e^{-\left(\log(S/X)+(r+\sigma^2/2)T\right)^2/(2\sigma^2 T)}, \tag{1.1}$$

where T is the time until option expiration, r is the interest rate, X is the strike price, σ is the underlying volatility and S is the value of the firm. Panel (b) shows the vega of the convertible preferred equity position.

Figure 1 shows that when the volatility is relatively low the direction of the volatility exposure is ambiguous. However, when the volatility is high enough, and in particular is the level observed for VC-backed firms, the negative volatility exposure of the concave part of the payoff is potentially larger than that from the convex part. This implies that this particular contractual feature of VC investments is unlikely to drive the positive volatility exposure of VC returns.

1.2.2. A Simple Model of Idiosyncratic Volatility and Reinvestment

An alternative potential driver of volatility exposure of VC investments lies in the (real) reinvestment option, or the contractual right of participation in future investment rounds.

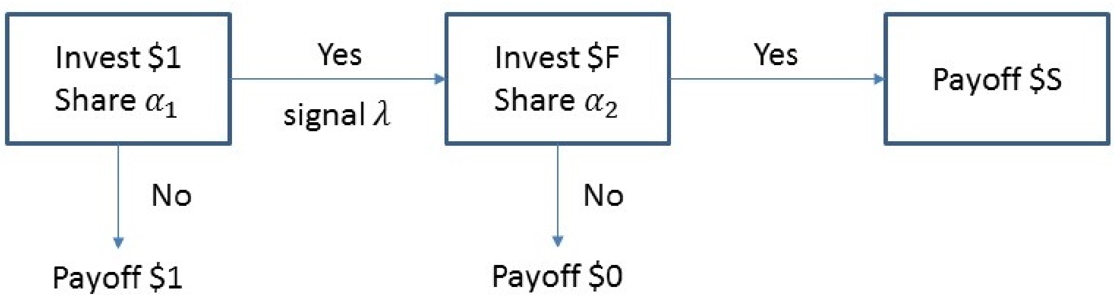
[5] A number of theoretical papers rationalize this form of equity participation by investors, eg. Cornelli and Yosha (2003), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), and Biais, Mariotti, Plantin and Rochet (2007)

[6] For purposes of this figure the option is assumed to be three years from expiration, the assumed dividend yield and interest rate are zero.

The contractual right of first refusal on future investment rounds takes the following form in the NVCA's 2013 model term sheet:

> "All Major Investors shall have a pro rata right, based on their percentage equity ownership in the Company (...), to participate in subsequent issuances of equity securities of the Company (...). In addition, should any Major Investor choose not to purchase its full pro rata share, the remaining Major Investors shall have the right to purchase the remaining pro rata shares."

I present a simple 3-period investment model with a risk-neutral, competitive investor and a penniless entrepreneur. The timing is as follows. In period 1, the entrepreneur decides whether to invest \$1 in the project. If he invests he receives a share α_1 of the project and receives a publicly observed signal (λ) about the project payoff. In period 2 he can either invest an amount F or not. If he decides not to invest the project dies (default) and he receives a payoff of zero. If he invests he receives a share α_2 of the project which dilutes his original stake. In period three the project payoff (S) is realized.



In the simplest construction of this model, the signal λ is discrete and all information is public. The payoff is $S \in \{S^L, S^H\}$ where $S^L = \mu - \sigma$ and $S^H = \mu + \sigma$ with equal probabilities. The public signal λ is informative about the type of the project:

$$P[\lambda = L|S = S^L] = P[\lambda = H|S = S^H] = \gamma$$

The high payoff is more likely with a high signal:

$$P[S = \mu + \sigma | \lambda = H] = \gamma$$

where, without loss of generality, $\gamma \in (0.5, 1]$ and a higher γ is associated with a more informative signal. Then the expected payoff is

$$E[S|\lambda = H] = \gamma(\mu + \sigma) + (1 - \gamma)(\mu - \sigma) = \mu - \sigma + 2\gamma\sigma$$
$$E[S|\lambda = L] = (1 - \gamma)(\mu + \sigma) + \gamma(\mu - \sigma) = \mu + \sigma - 2\gamma\sigma$$

Solving the model by backward induction, the share that leaves the competitive investor indifferent in the second period, if $\lambda = H$, is

$$\alpha_2^H = \frac{F}{\mu - \sigma + 2\gamma\sigma}$$

and when $\lambda = L$:

$$\alpha_2^L = \frac{F}{\mu + \sigma - 2\gamma\sigma}$$

Of course, investment is only feasible in the case that $\alpha_2 \in [0, 1]$. Investment in the second round, given a bad signal ($\lambda = L$), will only occur if $F < \mu - \sigma(2\gamma - 1) = \bar{F}$.

In period one, the investor invests one dollar for a share α_1, knowing this share will be diluted if he invests in the next round, in which case his final ownership share will be $(\alpha_2 + (1 - \alpha_2)\alpha_1)$. The share α_1 solves, in the case $F > \bar{F}$

$$1 = 0.5((1 - \alpha_2)\alpha_1)(\mu - \sigma + 2\gamma\sigma)$$
$$\Rightarrow \alpha_1 = \frac{2}{\mu + \sigma(2\gamma - 1) - F}.$$

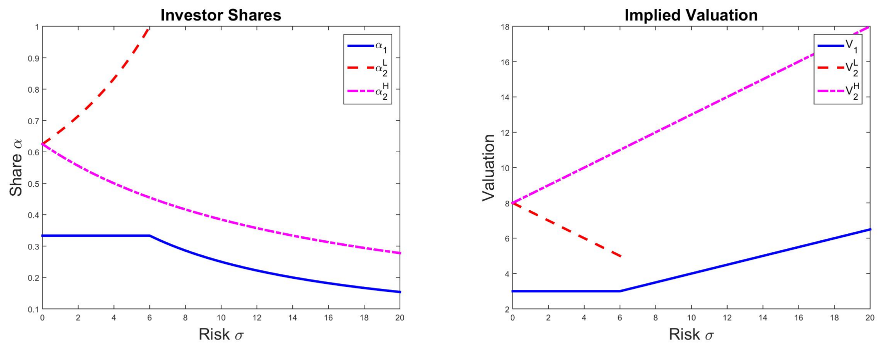
The share of the projects required as compensation for the initial investment (α_1) is decreasing in σ as long as $2\gamma - 1 > 0$, which is true by assumption.

In the alternative case $F < \bar{F}$, the share α_1 solves

$$
\begin{aligned}
1 &= 0.5((1-\alpha_2^H)\alpha_1)(\mu - \sigma + 2\gamma\sigma) + 0.5((1-\alpha_2^L)\alpha_1)(\mu + \sigma - 2\gamma\sigma) \\
\Rightarrow \alpha_1 &= \frac{1}{\mu - F}.
\end{aligned}
$$

Figure 2: Comparative Statics

This figure presents comparative statics for the model presented in Section 1.2.2. Panel (a) shows the shares that the investors receive for their investment as a function of the risk of the underlying cash flow while panel (b) shows the implied valuations that these shares represent.



(a) (b)

Figure 2 shows comparative statics for this simple model. Panel (a) shows the shares acquired in each period and state as a function of the risk (σ) of the underlying asset. Panel (b) shows the post-money valuation in each state as a function of the risk. It is easy to see that as long as reinvestment depends on the realization of the signal the value of the firm is increasing in the risk of the underlying asset.

Model Takeaways

The model formalizes the intuition that the real option to reinvest or abandon is valuable and that its value rises with the volatility of the underlying asset. Since the investor is perfectly competitive, the entrepreneur retains all of the surplus generated by the project, though in the real world one may expect either private information or other form of market power on the part of the investor to cause a more even split in surplus. Regardless, once the initial investment is made, any increase in the volatility of the asset σ will lead to an increased valuation for the investor.

Specifically, this simple model makes the time-series prediction that reinvestment options lead innovations to the level of asset volatility, after the initial investment and before the final investment, to increase investment performance. This prediction is distinct from that of, for example, Ewens, Jones and Rhodes-Kropf (2013) who investigate the implications of a principal-agent problem between investors and private equity firms. This friction drives a cross-sectional relationship between diversifiable risk and fund performance. These authors find that this relationship holds for both VC and buyout funds, as predicted by their model. Reinvestment options, in contrast, imply a time-series relationship only for VC firms, who retain reinvestment options, and not for buyout funds, who purchase going concerns.

1.3. Data Sources and Methodology

This section describes the data sources used in this paper. I use data on VC returns and investments at three levels of aggregation. First, I describe VC return indexes from Cambridge Associates (CA) and Sand Hill Econometrics (SHE), meant to proxy for the returns to investing in VC in aggregate. Second, I describe the private equity fund cash flow data from Burgiss. These are the realized cash flows experienced by investors into private equity funds. Third, I describe the investment-level data from the VentureSource database. These data describe the individual investments made by VC firms into their individual portfolio companies and the outcomes of these investments. I also describe construction of the measure of aggregate idiosyncratic volatility used throughout the paper and a measure of the level of idiosyncratic volatility of VC-backed portfolio companies.

1.3.1. Venture Capital Indexes

I use data on aggregate returns in the VC industry from Cambridge Associates and Sand Hill Econometrics. Cambridge Associates (CA) provides a quarterly net-of-fees VC returns series derived from disclosures by VC firm general partners. CA does not disclose what fraction of the universe of possible investments their data cover. One potential concern with the CA VC return index is that it is subject to asynchronous prices resulting from the fact that VCs infrequently update (mark to market) the value of their portfolio holdings. This is due to the fact that fair (market) value of privately held companies is only observed when a new transaction for shares in the portfolio company takes place, at which point the valuation of previously held shares is adjusted.[7] Those portfolio companies that do not experience a valuation event are left at the previous "stale" valuation. This reporting convention causes net asset values, and therefore the returns reported by Cambridge Associates, to appear smoother and less correlated with market returns than are the unobservable true returns. This asynchronous trading problem has been studied by, for example, Lo and MacKinlay (1990) and Boudoukh, Richardson and Whitelaw (1994). Risk factors in the presence of asynchronous prices can be recovered by projecting returns on contemporaneous and lagged factor returns and summing the estimated coefficients (Dimson 1979). Woodward (2009) contains a detailed example of this mechanism at work in VC data and estimates risk loadings in the CA VC indexes. Section 1.4.1 describes the quantitative effect of this correction. Korteweg and Sorensen (2010) propose an alternative correction which I discuss in Section 1.3.5.

I use five different return indexes provided by Cambridge Associates. The first is meant to proxy for returns in the VC industry as a whole. Three other indexes are meant to proxy for returns of VC funds with different investment strategies: early-, multi- and late-focused VC funds. The final index proxies for the returns to buyout funds.

Sand Hill Econometrics (SHE) provides a monthly gross-of-fees return series derived from the individual VC investments (VentureSource, discussed below) rather than the net asset values reported by VC funds. SHE makes an effort to remove any bias introduced by asynchronous prices by interpolating values between rounds, using market indications of change in values. Details on SHE index construction are in Blosser and Woodward (2014).

[7] In the VentureSource data, described below, the median (mean) time between consecutive financing rounds is 16 (21) months.

1.3.2. Venture Capital Fund Cash Flow Data

VC fund performance data are from Burgiss, a global provider of investment decision support tools for the private capital market, and are described in detail by Harris, Jenkinson and Kaplan (2014). The Burgiss dataset contains the complete transactional history for over 6,800 private capital funds with a total capitalization representing over $4.7 trillion in committed capital across the full spectrum of private capital strategies. Kaplan and Lerner (2016) report that the Burgiss dataset is the likely the best available dataset of its type, with a non-selected sample and very high coverage. The Burgiss dataset is representative of actual investor experience, as the data are sourced exclusively from limited partners, avoiding any reporting biases introduced by sourcing data from general partner surveys. I focus my analysis on a sample of 914 VC funds first raised before 2011. Of these funds, 513 are designated as early stage funds, 118 as late stage funds and the remaining 283 are designated as balanced funds. Cash flows include draw-downs, flows from investors to VC funds, as well as distributions, flows from funds back to investors.

The Public Market Equivalent

The public market equivalent ("PME"), introduced by Kaplan and Schoar (2005), is a measure that evaluates fund performance based on cash flows. As discussed in Sorensen and Jagannathan (2015), which this discussion largely follows, the PME provides a valid economic performance measure when the investor (the limited partner or "LP") has log-utility preferences and the return on the LPs total wealth equals the market return. When these conditions hold, the PME is a valid performance measure regardless of the risk of PE investments, and it is robust to variations in the timing and systematic risks of the underlying cash flows along with potential GP manipulations.

The PME calculation works as follows: Let $X(t)$ denote the cash flow from the fund to the LP at time t. This cash-flow stream is divided into its positive and negative parts, called distributions, $dist(t)$, and capital capital draw-downs, $draw(t)$. A distribution is a cash flow that is returned to the LP from the PE fund (net of fees) after the fund successfully sells a company. Capital draw-downs are the investments by the LP into the fund, including management fees. Distributions and draw-downs are then discounted using the realized market returns over the same time periods, and the PME is the ratio of the two resulting valuations:

$$PME = \frac{\sum_t \frac{dist(t)}{1+r_M(t)}}{\sum_t \frac{draw(t)}{1+r_M(t)}}.$$

The sum runs over the life of the fund and $r_M(t)$ is the realized market return. A PME greater than one suggests that the value of the distributions exceeds the cost of the capital calls, meaning the LP has benefited from the investment relative to the performance of the market.

1.3.3. Portfolio Company Data

Venture capital-backed company data are from the VentureSource database, maintained by Dow Jones. The full dataset contains 102,255 financing events for 30,689 companies, including seed, early, mezzanine and late round investments by VC firms, acquisitions by other companies and initial public offering (IPO) events. Most VC financings are syndicated, ie. involve more than one VC firm financing the company.

To calculate company-level investment returns requires data on exit valuation (either IPO or acquisition price) as well as the investment amount and fraction acquired for all rounds between the initial round and exit. Following the literature on investment-level VC returns, (e.g., Cochrane (2005), Korteweg and Sorensen (2010)), I calculate the gross multiple $M_{i,t}$ as:

$$M_{i,t} = \frac{V_i}{V_{i,t}^{Post}} \prod_{s=t+1}^{T} D_{i,s},$$

where T is the total number of dilutive financing rounds, V_i is the exit valuation, $V_{i,t}^{Post}$ is the post-money valuation for the round, and $D_{i,s}$ is the dilutive factor for each round, which is calculated as $1 - K_{i,s}/V_{i,s}^{Post}$, where $K_{i,s}$ is the total capital raised in financing round s.

As noted by Ewens, Rhodes-Kropf and Strebulaev (2016), investments that eventually have an initial public offering have a relatively higher probability of their valuation being reported. In contrast, acquisitions are much less likely to have prices and returns reported. As IPO returns tend to exceed those of acquisitions, this leads to positive selection in any VC returns data. Conversely, acquisition returns are underrepresented in the sample of observed returns. I address this concern by following the Korteweg and Sorensen (2010) approach of re-weighting the observed returns using the true exit weights in the full sample.

1.3.4. Public Market Idiosyncratic Volatility

One potential concern with using idiosyncratic volatility to explain VC investment returns and investment dynamics is the fact that the idiosyncratic volatility of VC-backed firms may be endogenously determined by the amount of VC investment through, for example, an experimentation channel as argued by Nanda and Rhodes-Kropf (2013, 2016). For this reason, I construct a measure from public market return data. Data on public equity market returns is from the Center for Research in Securities Prices (CRSP) as reported by Wharton Research Data Services (WRDS).

I construct a monthly measure of average idiosyncratic volatility that follows the measure documented in Herskovic et. al. (2016). The Common Idiosyncratic Volatility (CIV) is constructed using data from the daily CRSP stock file for the years 1975-2014. Idiosyncratic returns are constructed within each calendar month τ by estimating a factor model using all observations within the month and takes the form:

$$r_t^i = \gamma^i F_t + \varepsilon_t^i$$

where t denotes a daily observation in month τ. Idiosyncratic volatility for each firm is calculated as the variance of the residuals ε_t^i within each month. The return factor model is purely statistical and specifies F_t as a constant and the first five principal components of the cross section of returns within the month. CIV is calculated as the equally-weighted average idiosyncratic volatility across firms.

I deviate from Herskovic et. al (2016) by measuring idiosyncratic volatility shocks as statistical innovations from the following ARMA(1,1) model on the level of CIV

$$\sigma_t^2 = c + \varepsilon_t^{CIV} + \varphi\sigma_{t-1}^2 + \theta\varepsilon_{t-1}^{CIV},$$

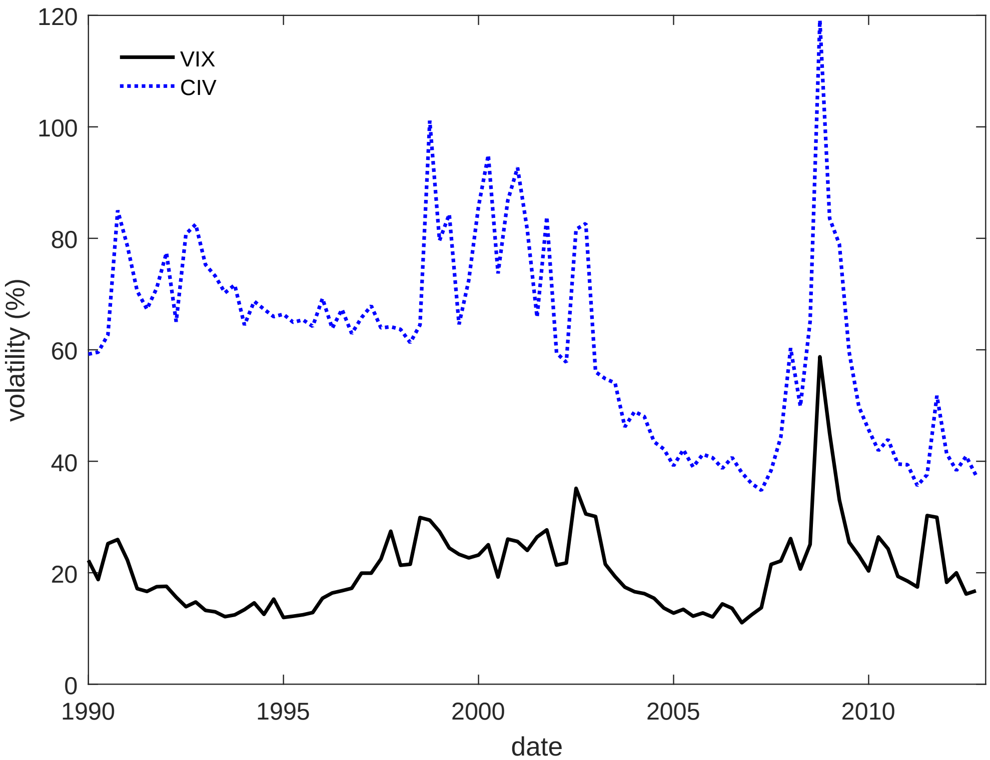
whereas Herskovic et. al. (2016) construct CIV shocks as first-differences in the level of CIV, assuming a unit root. I estimate that the autoregressive coefficient φ is 0.936 and statistically different from unity ($t = 3.85$) and the moving average coefficient θ is -0.090 ($t = -2.81$).

When regressions are at the quarterly or annual level, CIV shocks are measured as the average of the monthly CIV shocks over the quarter or year. Figure 3 shows quarterly CIV over time along with the VIX, a measure of systematic volatility. The correlation between the two series is 0.49 while the correlation between CIV shocks and similarly constructed VIX shocks is significantly lower (0.1).

1.3.5. Measuring Private Company Idiosyncratic Volatility

In order to validate the public market proxy (CIV) for the volatility of VC-backed firms, I compare low frequency estimates of the level of idiosyncratic volatility in these firms to similarly timed low frequency estimates from public markets. This section follows directly from Korteweg and Sorensen (2010); see their paper for details. These authors develop a Bayesian Markov Chain Monte Carlo estimation technique to estimate the risk and return characteristics of VC-backed company equity, incorporating a selection model to account for the fact that equity prices are observed infrequently and endogenously. This *dynamic selection problem* arises because valuations of these companies are observed only in the event that the company receives additional financing, either through a VC round or an initial public offering (IPO). They find that accounting for dynamic selection dramatically affects estimates of the market model parameters.

Figure 3: Observed CIV and VIX.



CIV is calculated as the equally-weighted average idiosyncratic volatility across firms from a five principal components market model. VIX is the CBOE Volatility Index, a measure of the implied volatility of S&P 500 index options. VIX is calculated by the Chicago Board Options Exchange (CBOE).

Korteweg and Sorensen (2010) address dynamic selection by simultaneously estimating the following two-equation model:

$$v(t) = v(t-1) + r + \delta + \beta(r_m(t) - r) + \varepsilon(t) \quad (1.2)$$

$$w(t) = Z'(t)\gamma_0 + v(t)\gamma_v + \eta(t) \quad (1.3)$$

where, in equation 1.2, $v(t)$ is the log valuation at time t, r is the risk-free rate, $r_m(t)$ is the market return and β is the factor loading on the market portfolio. They define

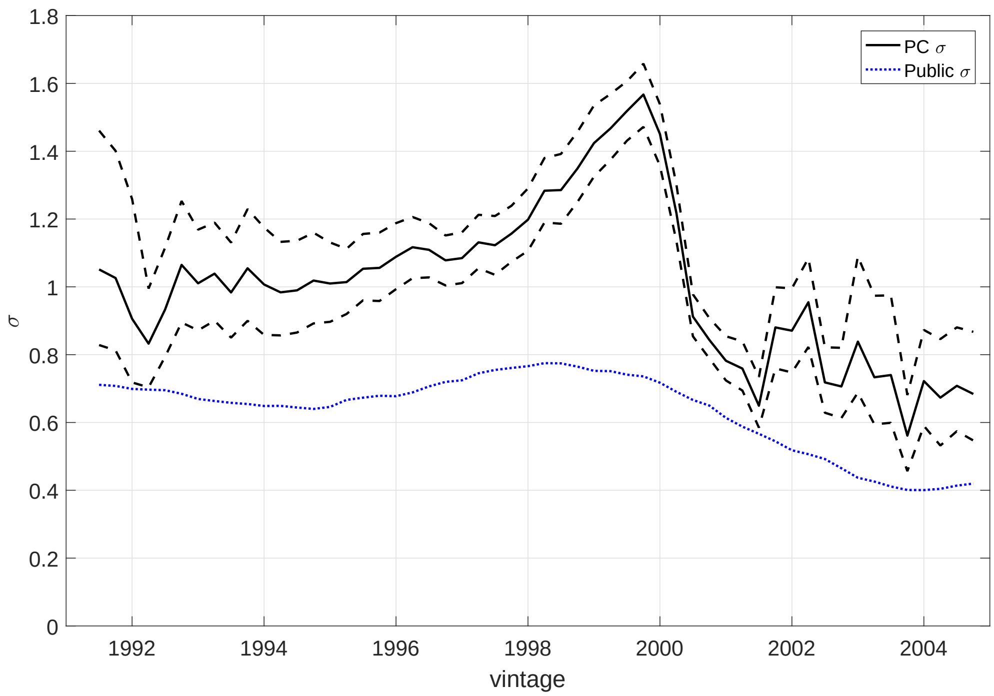
$$\delta = a - \frac{1}{2}\sigma^2 + \frac{1}{2}\beta(1-\beta)\sigma_m^2 \quad (1.4)$$

where σ and σ_m^2 are the variance of the asset and market return, respectively, and a is the excess return of the asset. It is easy to see that a simple rearranging of equation 1.2 delivers something similar to the usual capital asset pricing model (CAPM).

Valuations are only observed when a company has an event and equation 1.3 captures this selection process. Valuation is only observed when the latent selection variable $w(t)$ is greater than zero. The vector $Z(t)$ contains characteristics that affect refinancing and exit events and the error term $\eta(t)$ is normalized to have variance of one. The estimation technique uses a Bayesian Gibbs sampling procedure to estimate the parameters of the model δ, β, σ^2, γ_0 and γ_v.

I use this method to estimate the level of idiosyncratic volatility (σ) in VC portfolio companies over time. Figure 4 shows these estimates along with 4-year averages of CIV. The horizontal axis represents the cohort being studied, with some overlap to smooth out the estimates. For example, the 1996Q2 cohort was first financed between January and September 1996. I estimate the idiosyncratic volatility of these firms until exit. The solid line represents the annualized idiosyncratic volatility of this cohort and the dashed lines represent the Bayesian confidence interval. Since the median VC-backed firm exits in about 4 years, I compare these estimates with average CIV over 4 years starting in the same quarter. The

Figure 4: Low-frequency Volatility Estimates



This figure plots low-frequency estimates of idiosyncratic volatility measured in public (dotted black line) and private markets (solid blue line). Bayesian 95% posterior confidence intervals are denoted by dashed black lines. The estimates of public market idiosyncratic volatility are 4-year averages, staring at the date represented on the horizontal axis, of CIV as measured in public equity markets. The measure of private market idiosyncratic volatility is estimated on cohorts of portfolio companies using the Bayesian MCMC method of Korteweg and Sorensen (2010). A cohort is all VC-backed firms first financed in the three quarters around the date represented on the horizontal axis. The vertical axis is the annualized standard deviation of idiosyncratic volatility.

correlation between the two measures is 0.81, implying a substantial amount of commonality in the movement in idiosyncratic volatility between public and private markets. This result builds confidence in using a volatility measure derived from public equity markets.

1.4. Empirical Results

1.4.1. VC Industry Risk Exposures

I begin the empirical analysis by establishing that VC return indexes from two commonly used sources load on idiosyncratic volatility shocks. The first source is Cambridge Associates (CA) who report a number of net-of-fees VC indexes meant to benchmark for the VC industry as a whole as well as those VCs with specific investment focuses. CA also reports a benchmark index for buyout funds. The second source is Sand Hill Econometrics who report a gross-of-fees VC return index meant to proxy for the returns VC firms themselves earn from their portfolio companies. I then discuss the implications for expected returns of VC investments if the price of volatility risk is non-zero.

Net-of-fees VC Index from Cambridge Associates

Regression results using the net-of-fees VC return indexes from Cambridge Associates are presented in Table 1. To see the effect of asynchronous prices empirically, consider the naïve CAPM regression in column (1):

$$r_t^{CA} = \alpha + \beta_M (r_t^m - r_t^f) + \varepsilon_t.$$

We can see that the CAPM results imply that the market beta of VC is a small 0.47 and that the VC industry delivers a quarterly alpha of 2.58 percentage points. The problem with this result, as discussed in Section 1.3.1, is that the returns to VC as reported by CA are related both to contemporaneous and lagged market returns because of return smoothing and asynchronous prices. These features of VC portfolios lead to large serial correlation in the VC returns reported by CA.

Table 1: CIV Exposure of VC Industry Returns

Statistics are based on quarterly OLS regressions of venture capital industry returns, as reported by Cambridge Associates (CA), on contemporaneous and lagged excess market returns and CIV shocks:

$$r_t^{CA} - r_t^f = \alpha + \sum_\tau \beta_\tau (r_{t-\tau}^m - r_{t-\tau}^f) + \sum_\tau \delta_\tau \varepsilon_{t-\tau}^{CIV} + \varepsilon_t.$$

Excess returns of the VC indexes and the market are expressed in percentage points. The CIV shocks have been normalized to have a quarterly standard deviation of one. The row labeled $\sum\beta$ reports the sum of the market factor loadings and the row labeled $\sum\delta$ reports the sum of the CIV shock loadings. The row labeled F reports the p-value from a two-sided F-test for which the null hypothesis is that the sum of the coefficients on the CIV shocks are zero. In the first four columns the VC index used is for all VC funds. In columns (5), (6) and (7) the index is constructed using venture capital funds that CA reports as having an early, balanced and late investment focus, respectively. Column (8) reports an index constructed from buyout funds. Standard errors are in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CA	CA	CA	CA	Early	Balanced	Late	BO
r_t^m	0.468***	0.465***	0.507***	0.518***	0.546***	0.466***	0.411***	0.375***
	(0.094)	(0.093)	(0.093)	(0.094)	(0.108)	(0.073)	(0.071)	(0.037)
r_{t-1}^m		0.131	0.148	0.193**	0.249**	0.108	0.159**	0.107***
		(0.093)	(0.092)	(0.097)	(0.111)	(0.075)	(0.074)	(0.039)
r_{t-2}^m		0.223**	0.240**	0.310***	0.381***	0.186**	0.167**	0.080**
		(0.093)	(0.092)	(0.097)	(0.111)	(0.075)	(0.073)	(0.038)
r_{t-3}^m			0.214**	0.306***	0.343***	0.233***	0.271***	0.086**
			(0.091)	(0.096)	(0.109)	(0.074)	(0.073)	(0.038)
r_{t-4}^m			0.193**	0.285***	0.329***	0.203***	0.198***	0.087**
			(0.091)	(0.096)	(0.110)	(0.074)	(0.073)	(0.039)
r_{t-5}^m			0.090	0.173*	0.202*	0.153**	0.057	0.062
			(0.091)	(0.097)	(0.110)	(0.075)	(0.073)	(0.039)
ε_t^{CIV}				0.510	0.830	-0.077	0.128	-0.566*
				(0.834)	(0.954)	(0.646)	(0.622)	(0.316)
ε_{t-1}^{CIV}				0.504	0.784	-0.071	0.425	-0.363
				(0.836)	(0.956)	(0.647)	(0.621)	(0.318)
ε_{t-2}^{CIV}				1.884**	2.518***	0.654	0.884	-0.251
				(0.834)	(0.954)	(0.646)	(0.613)	(0.314)
ε_{t-3}^{CIV}				1.410*	1.673*	0.754	1.590**	0.162
				(0.832)	(0.951)	(0.644)	(0.616)	(0.315)
ε_{t-4}^{CIV}				2.504***	2.997***	1.656**	1.080*	0.265
				(0.830)	(0.949)	(0.642)	(0.616)	(0.318)
ε_{t-5}^{CIV}				0.945	0.975	0.776	0.705	0.284
				(0.805)	(0.921)	(0.623)	(0.592)	(0.302)
Constant	2.579***	1.899**	0.644	-0.182	-0.372	-0.027	0.660	1.851***
	(0.821)	(0.853)	(0.914)	(0.928)	(1.061)	(0.718)	(0.709)	(0.370)
$\sum\beta$	0.468	0.819	1.391	1.786	2.050	1.350	1.262	0.796
$\sum\delta$				7.757	9.776	3.691	4.813	-0.468
F				0.0003	0.0001	0.0242	0.0026	0.5606
R^2	0.156	0.204	0.279	0.383	0.378	0.396	0.378	0.631
N	136	134	131	123	123	123	119	107

We can address asynchronous prices by including lagged market returns in the regression as suggested by Dimson (1979):

$$r_t^{CA} = \alpha + \sum_{\tau} \beta_\tau (r_{t-\tau}^m - r_{t-\tau}^f) + \varepsilon_t.$$

For example, columns (2) and (3) of Table 1 report regression results including additional lags of the market return. The sum of the coefficient estimates is reported in the row labeled $\sum \beta$. Here we can see that the lagged market returns enter with positive and statistically significant coefficients, indicating that VC industry returns depend on contemporaneous and lagged market returns because of asynchronous prices. The sum of the betas, representing the true market exposure, is 1.39 and the risk-adjusted alpha is close to zero, once we account for asynchronous prices. The largest and most statistically significant *lagged* return coefficients are those on the 2-, 3- and 4-quarter lagged market returns.

To study the impact of idiosyncratic volatility shocks on VC returns, I additionally include contemporaneous and lagged CIV shocks to the regression:

$$r_t^{CA} - r_t^f = \alpha + \sum_{\tau} \beta_\tau (r_{t-\tau}^m - r_{t-\tau}^f) + \sum_{\tau} \delta_\tau \varepsilon_{t-\tau}^{CIV} + \varepsilon_t. \tag{1.5}$$

These shocks have been normalized to have a quarterly standard deviation of one. Column (4) of Table 1 reports results from this regression. The sum of the coefficient estimates on the CIV shocks is reported in the row labeled $\sum \delta$. We can see that past innovations in idiosyncratic volatility have a significant effect on VC returns as reported by CA. The sum of the coefficients on the idiosyncratic volatility shock in column (4) is 7.757, implying that a positive one-standard deviation shock to the level of idiosyncratic volatility corresponds to approximately an 8 percentage point return to VC investors. The row labeled “F” is the p-value from an F-test whose null hypothesis is that the sum of the coefficients on the CIV shock is zero, which we can easily reject at conventional levels. It is also interesting to note that the lagged CIV shocks with the largest and most statistically significant coefficients are those on the 2-, 3- and 4-quarter lagged shocks, similar to the lagged market return coefficients. The increase in the R^2 between columns (3) and (4) is 0.104, implying that CIV shocks explain a substantial portion of the return variance not explained by the market. Note that, once I include the idiosyncratic volatility shocks in the return regressions, the regression intercept can no longer be interpreted as a return, since the CIV shocks are statistical innovations and not tradeable portfolios. See Section 1.4.1 below for a discussion of the implications for risk-adjusted expected returns.

The remaining columns in Table 1 report results for other return indexes reported by Cambridge Associates. Columns (5), (6) and (7) report results for indexes constructed from early-, balanced- and late-focused funds, respectively. While all three VC strategy indexes load significantly on the CIV shocks, the loading for the early-focused fund index is much larger than that for the balanced- or late-focused fund indexes (9.78 versus 3.69 and 4.81, respectively). Early-focused VC funds also have a larger loading on the market return than late-focused funds. The final column in Table 1 reports results from the same regression where the dependent variable is now the benchmark return for leveraged buyout funds. In contrast to the results for VC indexes, the buyout index does not load significantly on CIV shocks.

To confirm that it is truly idiosyncratic volatility, rather than systematic volatility, driving the results I run untabulated regressions including contemporaneous and lagged measures of both CIV shocks and VIX shocks, where VIX shocks are measured as ARMA(1,1) innovations to the level of VIX. Shocks to the level of systematic volatility enter the regression insignificantly whether or not the regression includes CIV shocks. In contrast, shocks to CIV enter strongly significantly whether or not VIX shocks are included in the regressions.

Gross-of-fees VC Index from SHE

I also investigate the idiosyncratic volatility exposure of the VC return index from Sand Hill Econometrics (SHE). SHE provides a monthly, gross-of-fees VC return index that has been constructed to account for asynchronous prices as discussed in Blosser and Woodward (2014) and Woodward and Hall (2004). This series is constructed to measure the monthly return of a value-weighted portfolio of VC-backed companies, while the CA indexes discussed above are meant to represent averages of returns to investments in venture capital funds.

Factor regression results for this index are presented in Table 2. To establish a baseline for this index, column (1) shows the results for the Fama French 3-factor model. The SHE index has a large loading on the market return and a negative loading on the HML portfolio. While the contemporaneous CIV shock doesn't enter significantly (column (2)), lagged CIV shocks do seem to be significantly related to VC returns as measured by this index (Columns (3) and (4)). While only a few of the lagged CIV shocks are significant individually, their sum is large and highly significant. The summed value of the lags is 1.69, implying that a one standard deviation monthly CIV shock is associated with a 1.69% return. The row labeled "F" is again the p-value from an F-test whose null hypothesis is that the sum of the lagged CIV coefficients is zero.

The fact that the lagged exposure of the SHE index to CIV shocks is positive indicates that the SHE procedure does not purge VC investment returns of their lagged exposure to idiosyncratic volatility.

Implications for Expected Returns

The implications of volatility exposure for risk-adjusted VC returns depend on the price of idiosyncratic volatility risk. Herskovic et. al. (2016) present evidence showing that the level of idiosyncratic volatility proxies for idiosyncratic income risk faced by households and thus argue that the price of volatility risk is negative, i.e. that investors are willing to accept lower returns for assets have positive return exposure to aggregate idiosyncratic volatility. This is because such an exposure would hedge their own exposure to idiosyncratic income risk. In this case the inclusion of this priced risk will increase the risk-adjusted expected returns of VC investments.

To test this hypothesis, I run a regression like that in Eq. (1.5) but using a tradeable

Table 2: VC Industry Returns - Sand Hill Econometrics

Statistics are based on monthly OLS regressions of venture capital industry returns, as reported by Sand Hill Econometrics, on contemporaneous and lagged excess market and factor returns and contemporaneous and lagged CIV shocks:

$$r_t^{SHE} = \alpha + \beta_M (r_t^m - r_t^f) + \beta_S r_t^s + \beta_H r_t^h + \sum_\tau \delta_\tau \varepsilon_{t-\tau}^{CIV} + \varepsilon_t$$

Excess returns of the VC index, market and return factors are expressed in percentage points. The risk factors are the SMB and HML factors of Fama and French (1992). The CIV shocks have been normalized to have a monthly standard deviation of one. The row labeled F reports the p-value from a two-sided F-test for which the null hypothesis is that the sum of the coefficients on the CIV shocks are zero.

	(1)	(2)	(3)	(4)
Mkt-RF	1.564***	1.566***	1.577***	1.579***
	(0.039)	(0.039)	(0.039)	(0.040)
SMB	0.038	0.040	0.047	0.057
	(0.051)	(0.052)	(0.051)	(0.052)
HML	-0.664***	-0.661***	-0.625***	-0.627***
	(0.054)	(0.055)	(0.056)	(0.057)
ε_t^{CIV}		0.043		0.050
		(0.150)		(0.154)
ε_{t-1}^{CIV}			0.365**	0.378**
			(0.147)	(0.149)
ε_{t-2}^{CIV}			-0.137	-0.143
			(0.145)	(0.149)
ε_{t-3}^{CIV}			0.232	0.239
			(0.151)	(0.155)
ε_{t-4}^{CIV}				0.312**
				(0.150)
ε_{t-5}^{CIV}				0.185
				(0.148)
ε_{t-6}^{CIV}				0.157
				(0.150)
ε_{t-7}^{CIV}				0.120
				(0.149)
ε_{t-8}^{CIV}				0.122
				(0.149)
ε_{t-9}^{CIV}				0.016
				(0.152)
ε_{t-10}^{CIV}				-0.029
				(0.148)
ε_{t-11}^{CIV}				0.287*
				(0.148)
Constant	0.779***	0.777***	0.750***	0.717***
	(0.163)	(0.164)	(0.162)	(0.162)
$\sum \delta$			0.46	1.69
F			0.074	0.002
r2	0.892	0.892	0.896	0.900
N	274	274	274	274

portfolio that is exposed to the level of aggregate idiosyncratic volatility. I construct such a portfolio by taking an equally weighted position in 91-day straddle positions in S&P 500 constituents on the last trading day of each quarter. The construction of the option straddle portfolio is detailed in the appendix. This option portfolio delivers a mean return of -2.46% per quarter with a standard deviation of 21% for an annualized Sharpe ratio of -0.23. The average risk-adjusted return and Sharpe ratio are virtually unchanged in a Carhart (1997) four-factor model. Regression results are in Table 3. Since option prices on individual stocks are only available beginning in 1996, there are many fewer observations.

I start by re-running the analysis of Table 1 column (3) in column (1) of Table 3 to give a baseline for the estimates of market slope and intercept, which are both somewhat higher in this time period, relative to the estimates from the longer time period in Table 1. Column (2) adds the contemporaneous and lagged returns on the option straddle portfolio. The sum of coefficient estimates is a very statistically significant 0.70. Since the explanatory variables are all market returns, the intercept of the regression can be interpreted as a risk-adjusted excess return. Including the option straddle portfolio increases the intercept to a marginally significant 2.52% per quarter, which corresponds to an annualized increase of about 6% per year. In unreported regressions, I find that if CIV shocks are included in the regression along with returns on the option straddle portfolio, the loadings on the option portfolio returns become insignificant while the loadings on the CIV shocks remain quite significant. This suggests that whatever explanatory power the option portfolio has is fully subsumed by the actual idiosyncratic volatility shocks.

Columns (3)-(5) repeat the analysis for early-, balanced- and late-focused VC funds respectively. The patterns are similar to those in Table 1 in that the coefficient estimates on the idiosyncratic volatility proxy are largest for the early-focused VC fund benchmark. Column (6) repeats the analysis for the buyout fund benchmark, which does not load significantly on the option portfolio return.

Table 3: Option Portfolio Exposure of VC Industry Returns

Statistics are based on quarterly OLS regressions of venture capital industry returns, as reported by Cambridge Associates (CA), on contemporaneous and lagged excess market returns and straddle option portfolio returns:

$$r_t^{CA} - r_t^f = \alpha + \sum_\tau \beta_\tau (r_{t-\tau}^m - r_{t-\tau}^f) + \sum_\tau \delta_\tau r_{t-\tau}^{opt} + \varepsilon_t.$$

Excess returns of the VC indexes and the market are expressed in percentage points. The straddle option portfolio is constructed by taking an equally weighted straddle (long call and long put) position in each of the S&P 500 constituents. The row labeled $\sum \beta$ reports the sum of the market factor loadings and the row labeled $\sum \delta$ reports the loading on the straddle option portfolio return. The row labeled F reports the p-value from a two-sided F-test for which the null hypothesis is that the sum of the coefficients on the straddle option portfolio return are zero. In the first two columns the VC index used is for all VC funds. In columns (3), (4) and (5) the index is constructed using venture capital funds that CA reports as having an early, balanced and late investment focus, respectively. Column (6) reports an index constructed from buyout funds. Standard errors are in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
	CA	CA	Early	Balanced	Late	BO
r_t^m	0.62***	0.59***	0.63***	0.52***	0.50***	0.46***
	(0.154)	(0.145)	(0.163)	(0.114)	(0.097)	(0.042)
r_{t-1}^m	0.19	0.18	0.20	0.16	0.17*	0.15***
	(0.152)	(0.143)	(0.160)	(0.112)	(0.096)	(0.041)
r_{t-2}^m	0.26*	0.26*	0.31*	0.19*	0.12	0.10**
	(0.152)	(0.144)	(0.161)	(0.113)	(0.096)	(0.041)
r_{t-3}^m	0.26*	0.26*	0.28*	0.23**	0.27***	0.09**
	(0.152)	(0.145)	(0.162)	(0.113)	(0.097)	(0.041)
r_{t-4}^m	0.24	0.25*	0.30*	0.18	0.07	0.04
	(0.152)	(0.146)	(0.163)	(0.114)	(0.098)	(0.042)
r_{t-5}^m	0.10	0.08	0.10	0.08	0.03	0.02
	(0.153)	(0.146)	(0.163)	(0.114)	(0.098)	(0.042)
r_t^{opt}		0.05	0.07	0.02	0.05	-0.03
		(0.070)	(0.079)	(0.055)	(0.047)	(0.020)
r_{t-1}^{opt}		0.09	0.12	0.03	0.04	-0.02
		(0.070)	(0.079)	(0.055)	(0.047)	(0.020)
r_{t-2}^{opt}		0.11	0.15*	0.04	0.06	0.01
		(0.071)	(0.080)	(0.056)	(0.047)	(0.020)
r_{t-3}^{opt}		0.19**	0.23***	0.10*	0.07	-0.00
		(0.071)	(0.080)	(0.056)	(0.048)	(0.020)
r_{t-4}^{opt}		0.13*	0.15*	0.06	0.06	0.00
		(0.070)	(0.079)	(0.055)	(0.047)	(0.020)
r_{t-5}^{opt}		0.13*	0.16**	0.08	0.03	-0.03
		(0.068)	(0.077)	(0.054)	(0.046)	(0.020)
Constant	0.88	2.52*	3.09*	1.06	1.94*	1.63***
	(1.544)	(1.495)	(1.675)	(1.172)	(1.001)	(0.429)
$\sum \beta$	1.675	1.634	1.822	1.364	1.155	0.867
$\sum \delta$		0.697	0.880	0.340	0.316	-0.058
F		0.0003	0.0001	0.0191	0.0111	0.2686
R^2	0.313	0.464	0.478	0.442	0.468	0.744
N	70	70	70	70	70	70

1.4.2. Investment Sample Results

To understand the impact of idiosyncratic volatility on individual VC investments, I next use the cash flows from individual VC funds. VC fund cash flow data has two large advantages over the industry indexes studied above. First, the funds are investment vehicles and the reported cash flows are the actual cash flows that investors receive. Given the illiquid nature of private equity funds, it is important to take the timing of cash flows into account. The second large benefit of using individual fund cash flows is that it allows an investigation into when, during a funds life, asset volatility shocks matter the most. This will in turn allow a better understanding of the mechanism driving this unique risk exposure.

I begin by regressing the PME on the average level and innovations to idiosyncratic volatility. The basic empirical specification is as follows:

$$PME_i = \alpha + \beta CIV_i + \epsilon_i \tag{1.6}$$

where PME_i is calculated from quarterly fund cash flows and CIV_i is a measure of the level of or innovation in CIV scaled by the cross sectional standard deviation. Results are in Table 4. Panel A reports results for regressions of PME on the average level of CIV over the fund life, from the month the fund was founded through the last cash flow has been distributed, an average of ten years. In the first column, labeled "VC", the sample is VC funds covered by Burgiss. The positive and statistically significant coefficient implies that funds earn higher returns relative to public market equity when the level of CIV is higher over the life of the fund. The coefficient of 0.204 implies that funds that experience lifetime idiosyncratic volatility one standard deviation above (the cross-sectional) average outperform public markets by over 20% more than average over the course of the fund's life.

The other columns of panel A investigate the same regression in subsamples. VC funds in Burgiss are designated as early-, late- or balanced-focus funds, depending on what stage

Table 4: Investment Outcomes

Statistics are based on OLS regressions of the Public Market Equivalent (PME) of Kaplan and Schoar (2005) on CIV levels and shocks:

$$PME_i = \alpha + \beta CIV_i + \varepsilon_i.$$

The variable PME is calculated by discounting the actual cash outflows and cash inflows that the fund received with the returns on publicly traded equity over the same time period and forming the ratio of the discounted cash inflows over the discounted outflows. Fund cash flow data are from Burgiss. The sample for the column labeled "VC" is all VC firms. The sample for columns "Early", "Balance" and "Late" are VC funds noted as having early, balanced and late stage focus, respectively. The sample for the column labeled "Buyout" is all buyout funds. Panel A reports coefficients for regressions of PME on the average level of CIV over the fund life. The independent variable in Panel B is the average CIV shock over the fund life and in Panel C is the average CIV shock calculated between months 12 and 48 of the fund life. Standard errors, in parentheses, are clustered by VC firm and are robust to heteroscedasticity.

Panel A: Regressions on Average CIV

	VC	Early	Balance	Late	Buyout
average CIV	0.204***	0.275**	0.124*	0.037	0.029
	(0.057)	(0.084)	(0.049)	(0.132)	(0.027)
Observations	914	513	283	118	616
R^2	0.013	0.017	0.01	0.001	0.003

Panel B: Regression on average CIV Shock

	VC	Early	Balance	Late	Buyout
$\bar{\varepsilon}^{CIV}$	0.176***	0.238***	0.095*	0.051	0.021
	(0.045)	(0.065)	(0.042)	(0.122)	(0.025)
Observations	914	513	283	118	616
R^2	0.011	0.014	0.006	0.001	0.001

Panel C: Regression on early CIV Shock

	VC	Early	Balance	Late	Buyout
$\bar{\varepsilon}^{CIV}$	0.394***	0.531***	0.270***	0.101	-0.004
	(0.061)	(0.097)	(0.075)	(0.110)	(0.018)
Observations	914	513	283	118	616
R^2	0.044	0.056	0.045	0.007	0.000

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

their portfolio companies are in when the funds first invest. We should expect idiosyncratic volatility exposure to be largest for those VC funds that invest in early-stage investments, since these funds are more likely to have reinvestment opportunities. The other columns in panel A show that this is indeed the case: the coefficient on average CIV for early stage funds is larger and more significant than for the other fund types. Late stage funds have an insignificant loading on average CIV, as do buyout funds.

Panels B and C of Table 4 show results for regressions of PME on average CIV shocks. CIV shocks are calculated as documented in section 1.3.4. In panel B, CIV shocks are averaged over the life of the fund. Results using CIV shocks averaged over the fund life are similar to those using level, which is reasonable considering the level reflects accumulated shocks. In particular, the pattern of early stage-focused VC fund performance being more exposed to idiosyncratic volatility remains. In unreported regressions, I include the initial level of CIV at fund initiation. This additional explanatory variable enters insignificantly, implying that any information about future expected volatility contained in the level is fully incorporated into the price of fund investments.

Private equity funds usually last for about 10 years. Therefore the idiosyncratic volatility shocks that I have investigated so far are averaged over those very long time periods. Panel C of Table 4 investigates the impact of idiosyncratic volatility shocks averaged over the 12th through 48th month of the fund's life, the time when reinvestments are most likely to happen. The coefficient estimates and R^2 are significantly higher when we focus on this period of the fund life. Focusing on the full VC sample (column 1), I find that funds that experience idiosyncratic volatility one standard deviation above (the cross-sectional) average in years two through four of fund life outperform public markets by almost 40% more that average over the course of the fund's life. This rises to 53% when we consider the subsample of early-focused funds. This is equivalent to an annual return of 4.3% over a ten year fund. The R^2 for this sample implies that the time-series of CIV shocks explains over 5% of the cross-sectional variance of early-focused VC funds.

For a more granular look, I next regress VC fund performance on multiple CIV shocks, one for each of the first seven years the fund was in operation. To build these shocks, I average the monthly CIV shock across each fund year. For example, if a fund was first raised in June 1995, the first CIV shock is measured as the average monthly CIV shock from July 1995 through June 1996, the second is from July 1996 through June 1997, etc. The explanatory variables are again scaled so that one is equivalent to a single cross-sectional standard deviation. Results are in Figure 5 which shows coefficient estimates and 95% confidence bounds on these estimates. We can see that the impact of CIV shocks starts positive and marginally statistically significant, is highest and highly significant in years 3 and 4 of the funds life, and falls close to zero by year 6. Together these results suggest that CIV shocks help explain VC fund performance and that this effect is stronger during the period when VC funds are likely to have already made investments and to be exercising reinvestment options.

I next ask what the implication of these fund-level regression results is for the time-series of VC fund performance. Using the same Burgiss dataset that underlies this section, Harris, Jenkinson and Kaplan (2014) find that "Average VC fund returns in the United States [...] outperformed public equities in the 1990s but have underperformed public equities in the most recent decade." Korteweg and Sorensen (2010) document a similar pattern in the returns of VC-backed companies. Accounting for *ex-post* realized shocks to the aggregate level of idiosyncratic volatility can help to explain these patterns.

Figure 6 shows mean and median fund performance, as measured by PME, along with the predicted performance given realized CIV shocks. The dashed (red) line is the mean fund performance and the dotted (blue) line is the median fund performance for funds within each vintage cohort. The mean is generally higher than the median due to the skewed performance of VC funds. Vintage year is defined by the date of the first fund cash flow. These realized performance measures, solid black line, are matched with fitted performance measures from regression Equation 1.6 where the measure of CIV used is average over the

Figure 5: Coefficients of regression of PME on averages of CIV shocks.

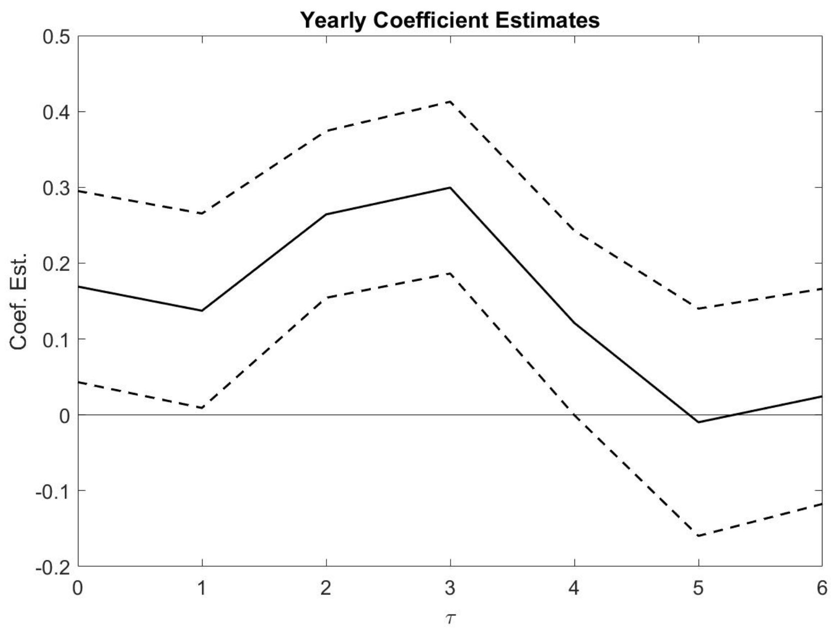
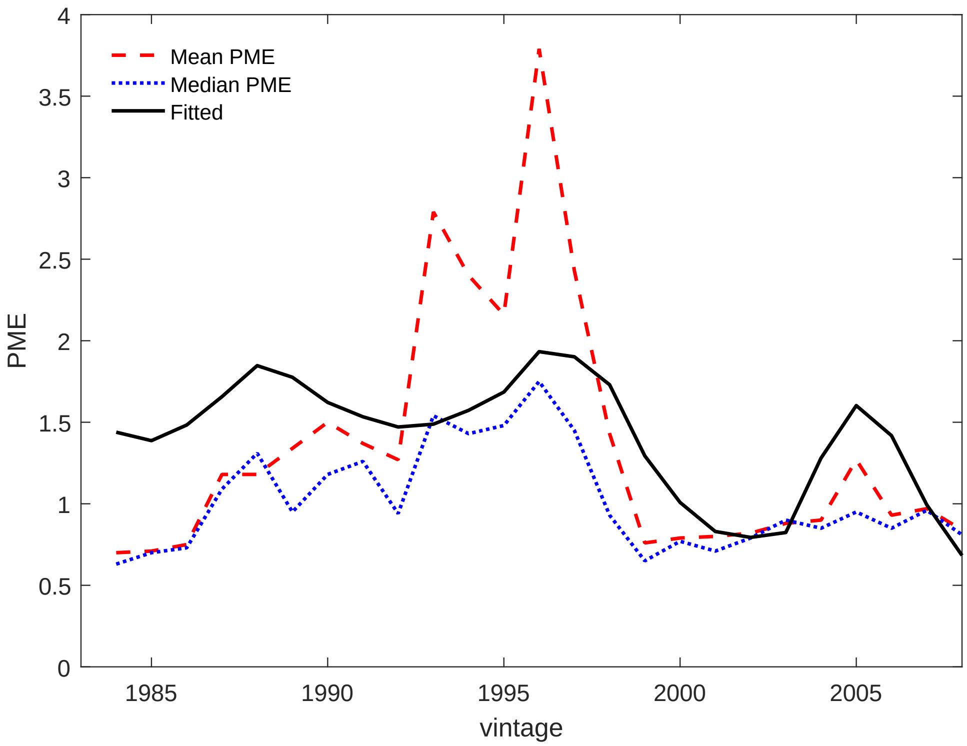


Figure reports coefficient estimates and confidence bounds from the following regression:

$$PME_i = \alpha + \sum_{\tau=0}^{6} \beta CIV_{i,t+\tau} + \epsilon_i$$

where PME_i is the public market equivalent of firm i as measured using the method of Kaplan and Schoar (2005) from fund cash flows and $CIV_{i,t+\tau}$ is the average (over monthly observations) CIV shock in year $t + \tau$ where year t is the year of fund formation. Standard errors are clustered by venture capital firm and robust to heteroscedasticity.

Figure 6: Average and predicted performance of VC funds by vintage.



Vintage years are defined by the date of the first fund cash flow. Performance is measured as the public market equivalent (PME) of Kaplan and Schoar (2005) from Burgiss fund cash flows. The dashed (red) line is the mean PME of funds first raised in each vintage year. The dotted (blue) line is the median PME. The solid line is the (ex-post) predicted average performance of VC funds in each year conditional on future realizations of CIV shocks.

12th through 48th month as in Panel C of Table 4. The fitted performance demonstrates that, given the *ex-post* realized CIV shocks, the difference in relative performance over time is not as large as previously believed.

Alternative β benchmarks

One potential concern with the PME measures introduced in section 1.3.2 and used in section 1.4.2 is that the relevant benchmark for VC funds may not be a market index with a β of unity. To address this concern, I repeat the analysis or section 1.4.2 using market benchmarks with different levels of market exposure. Results are in Table 5 and indicate that the broader pattern remains no matter what β benchmark is used.

1.4.3. Portfolio Company Sample Results

The final data set I investigate is Dow Jones VentureSource, discussed in Section 1.3.2. I focus on a sample of 111,766 investments by 1,615 VC firms into 19,638 individual portfolio companies made between 1990 and 2011. There are 47,411 financing events, most of which involve more than one investor. I require a VC firm to participate in at least ten financing rounds to enter the sample. I match these financing events with public market condition. Public market data are from the web page of Kenneth French and CRSP. Corporate accounting data are from Compustat. To measure average market prices of public firms I calculate an average measure of Tobin's q that incorporates intangible assets in the denominator from Peters and Taylor (2016). Market returns ($r^m_{t,t+1}$) are value weighted and measured over the following year and are calculated using the CRSP universe. Summary Statistics are presented in Table 6.

The variables IPO and DEF are indicators for whether the company has had a successful IPO or default respectively. A company is identified as having defaulted if it is listed as out of business, in bankruptcy, or has been otherwise identified by VentureSource as not active. A company is also labeled as having defaulted if it received the first VC funding prior to 2004, has yet to exit by the end of the sample, and has never had a successful

Table 5: PME Exposures for Different β Benchmarks

Statistics are based on OLS regressions of the Public Market Equivalent (PME) of Kaplan and Schoar (2005) on CIV shocks. The variable PME is calculated by discounting the actual cash outflows and cash inflows that the fund received with the returns on publicly traded equity over the same time period and forming the ratio of the discounted cash inflows over the discounted outflows. Fund cash flow data are from Burgiss. CIV shocks are calculates as the 12-month average of monthly CIV shocks measured relative to fund founding. For example, if a fund is established in July of year t, $\varepsilon^{CIV}_{t,t+1}$ is the average of the monthly CIV shocks from that July through the following June. Standard errors, in parentheses, are clustered by VC firm and are robust to heteroscedasticity.

	(1) PME(1.0)	(2) PME(1.5)	(3) PME(2.0)	(4) PME(2.5)	(5) PME(3.0)
$\varepsilon^{CIV}_{t,t+1}$	0.223**	0.213**	0.211***	0.219***	0.244***
	(0.074)	(0.066)	(0.062)	(0.062)	(0.066)
$\varepsilon^{CIV}_{t+1,t+2}$	0.101	0.093	0.090	0.092	0.104
	(0.075)	(0.067)	(0.063)	(0.063)	(0.068)
$\varepsilon^{CIV}_{t+2,t+3}$	0.304***	0.291***	0.286***	0.292***	0.311***
	(0.065)	(0.058)	(0.055)	(0.054)	(0.058)
$\varepsilon^{CIV}_{t+3,t+4}$	0.345***	0.315***	0.300***	0.301***	0.321***
	(0.066)	(0.059)	(0.055)	(0.055)	(0.059)
$\varepsilon^{CIV}_{t+4,t+5}$	0.159*	0.122	0.092	0.067	0.045
	(0.070)	(0.062)	(0.058)	(0.058)	(0.063)
$\varepsilon^{CIV}_{t+5,t+6}$	0.012	-0.044	-0.091	-0.139	-0.193*
	(0.089)	(0.079)	(0.074)	(0.074)	(0.079)
$\varepsilon^{CIV}_{t+6,t+7}$	0.026	-0.009	-0.038	-0.067	-0.098
	(0.083)	(0.074)	(0.070)	(0.069)	(0.075)
Constant	1.208	1.056	0.965	0.926	0.940
	(0.074)	(0.065)	(0.061)	(0.061)	(0.066)
R^2	0.073	0.073	0.074	0.077	0.079
Observations	834	834	834	834	834

Table 6: Summary Statistics: Individual VC Investments

Statistics are based on the VentureSource dataset. The sample covers venture capital investments bade between 1990 and 2011. IPO and DEF are indicator variables for whether the company has a successful IPO or observed default, respectively. CIV_t is the CIV at the round close date. $\varepsilon_{t,t}^{CIV}$ is the CIV shock over the next year. CIV levels and shocks have been normalized to have mean zero and standard deviation of one. Indus Q is the average Tobin's q in publicly traded hi-tech firms at the round close date. $r_{t,t+1}^{m}$ is the market return over the following year. Supply is the dollars invested in VC funds during the quarter as reported by the NVCA. lage is the log age at first financing. syndsize is the size of the investing syndicate. lraised is the log dollars raised by the company in its first round. cali and mass are indicator variables indicating whether the company is based in CA or MA. Multiple is the cash multiple of money earned by a hypothetical dollar invested in the financing accounting for any future dilution (zero for failed firms). Ann Ret is the multiple divided by the time between the first rounds and exit, unavailable for failed firms. If the time between the round and exit is less than one, the cash multiple is used.

variable	**mean**	**sd**	**min**	**p25**	**p50**	**p75**	**max**	**N**
IPO	0.155	0.362	0.000	0.000	0.000	0.000	1.000	111766
DEF	0.289	0.453	0.000	0.000	0.000	1.000	1.000	111766
early	0.292	0.455	0.000	0.000	0.000	1.000	1.000	111766
CIV_t	0.000	1.000	-1.487	-0.978	0.069	0.853	2.985	111766
$\varepsilon_{t,t+1}^{CIV}$	0.000	1.000	-2.136	-0.828	0.038	0.497	3.200	111766
induq	2.072	0.924	0.702	1.465	1.842	2.437	5.242	111766
$r_{t,t+1}^{m}$	0.010	0.196	-0.568	-0.138	0.063	0.141	0.439	111766
supply	9.895	0.926	7.602	9.354	10.129	10.345	11.527	111766
lage	6.821	1.309	0.000	6.358	7.074	7.617	10.494	111766
syndsize	3.706	2.355	1.000	2.000	3.000	5.000	20.000	111766
lraised	2.001	1.216	-4.605	1.281	2.079	2.826	7.313	107632
cali	0.454	0.498	0.000	0.000	0.000	1.000	1.000	111766
mass	0.129	0.336	0.000	0.000	0.000	0.000	1.000	111766
Multiple	3.825	7.514	0.000	0.000	1.438	3.870	48.736	20780
Ann Ret	1.258	3.305	-1.000	0.066	0.443	1.308	47.736	14669

exit event. I designate a round as an early stage investment if VentureSource identifies the round as being either a seed, angel or first round and a late stage round otherwise, 29.2% of the sample. The variable 'supply" is the quarterly flow of capital into the VC sector as reported by the National Venture Capital Association (NVCA). In all regressions in this section, CIV_t and Q_t are measured at the date of financing, $\varepsilon^{CIV}_{t,t+1}$ is the sum of monthly CIV shocks over the year following the first financing and $r^m_{t,t+1}$ is the market return over this same period.

I begin my analysis of company-level data with a consistency check: do firms funded in times of high idiosyncratic volatility experience more extreme investment outcomes? To investigate this question I identify extreme outcomes as a firm having either an observed default or an IPO. Table 7 regresses these firm outcomes on various firm characteristics and aggregate conditions. The dependent variable in columns (1)-(3) is an indicator for firm default, and coefficient estimates are expressed in percentage points. Column (1) regresses this variable on the level of CIV at the first financing round, the Industry Q of small, high-tech firms in the quarter of financing, and innovations to both the level of CIV and market price. The coefficient estimates indicate that companies first financed in times of high prices and high idiosyncratic volatility are more likely to default. Column (2) includes a number of company-level control variables: The firm age at first financing as reported by VentureSource, the size of the investing syndicate, the log dollars raised and whether the company is based in California or Massachusetts. Including these variables does not qualitatively impact the result. Column (3) adds industry fixed effects to the regression, which does not change the results appreciably.

Columns (4)-(6) of Table 7 studies the effect of market conditions at first financing on firms' probability of successfully completing an IPO. This is generally considered the best outcome for a VC-backed company. The dependent variable is an indicator for whether the firm has a successful IPO, again expressed in percentage points. Column (4) shows that firms first financed in high CIV times are more likely to successfully IPO. Interestingly, average stock

Table 7: Investment Outcomes

This table reports results from OLS regressions looking at the probability of various firm outcomes. The dependent variable in columns (1)-(3) takes a value of one if the firm fails and zero otherwise. The dependent variable in columns (4)-(6) takes a value of one if the firm has a successful IPO and zero otherwise. Industry fixed effects control for seven industries. Standard errors, clustered by quarter, in parentheses: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

	Prob. of Default			Prob. of IPO		
	(1)	(2)	(3)	(4)	(5)	(6)
CIV_t	3.79***	3.35**	3.50**	1.00***	1.01***	1.02***
	(1.32)	(1.36)	(1.39)	(0.26)	(0.27)	(0.26)
Indus. Q	6.33***	7.06***	7.45***	-0.37	-0.56**	-0.48*
	(1.48)	(1.51)	(1.52)	(0.25)	(0.26)	(0.26)
$\varepsilon^{CIV}_{t,t+1}$	0.20	0.13	-0.04	0.62**	0.68**	0.73**
	(1.37)	(1.43)	(1.44)	(0.28)	(0.30)	(0.29)
$r^{m}_{t,t+1}$	0.92	0.06	0.50	0.51	0.86	1.13
	(7.20)	(7.15)	(7.31)	(1.48)	(1.55)	(1.52)
supply	0.13	0.62	0.71	-2.58***	-2.62***	-2.47***
	(1.35)	(1.40)	(1.44)	(0.40)	(0.42)	(0.41)
lage		-0.49*	-0.52**		0.14***	0.17***
		(0.25)	(0.26)		(0.05)	(0.05)
syndsize		0.07	0.14		0.53***	0.45***
		(0.49)	(0.48)		(0.17)	(0.17)
lraised		-3.86***	-3.77***		0.78***	0.81***
		(0.55)	(0.54)		(0.16)	(0.15)
cali		-2.92***	-2.04**		0.28	0.52**
		(0.99)	(0.97)		(0.23)	(0.24)
mass		-6.72***	-5.79***		0.26	0.32
		(1.63)	(1.65)		(0.36)	(0.36)
Industry F.E.	N	N	Y	N	N	Y
r2	0.030	0.042	0.050	0.018	0.022	0.027
N	11182	10370	10370	11182	10370	10370

prices (at time of first financing) do not seem to greatly impact the probability that VC-backed firms successfully IPO, implying that the increase observed in columns (1)-(3) comes primarily from a decrease in the number of firms acquired. Finally, firms first financed in times of high VC fund supply are markedly less likely to successfully IPO. Including additional firm-level controls and industry fixed effects, as in columns (5) and (6) do not change these results.

Table 8: Frequency of Reinvestment

Statistics are based the VentureSource sample of venture capital investments. The dependent variable is an indicator variable for whether the portfolio company has another investment from the same venture capital firm. Columns (1)-(3) use the full sample of investments. Columns (4)-(6) use the subsample of early investments. Columns (7)-(9) use the subsample of late investments. Early investments are those that VentureSource designates as seed, angel or first round investments made by a venture capital firm. Standard errors, clustered by quarter, in parentheses: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

	All Rounds			Early Rounds			Late Rounds		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
CIV_t	-0.01*	-0.01**	-0.01**	0.01	0.01	0.00	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)
Indus. Q	-0.00	-0.01**	-0.01**	0.01	0.00	-0.00	-0.02***	-0.03***	-0.02***
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)
$\varepsilon^{CIV}_{t,t+1}$	0.02***	0.01*	0.01**	0.04***	0.04***	0.04***	0.00	-0.01	-0.00
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$r^m_{t,t+1}$	0.20***	0.15***	0.16***	0.31***	0.30***	0.31***	0.10***	0.06	0.07*
	(0.03)	(0.03)	(0.03)	(0.06)	(0.05)	(0.05)	(0.03)	(0.04)	(0.04)
supply	-0.00	-0.01	-0.00	-0.02**	-0.02**	-0.01	0.00	-0.00	0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
lage		-0.06***	-0.06***		-0.03***	-0.03***		-0.09***	-0.09***
		(0.00)	(0.00)		(0.00)	(0.00)		(0.01)	(0.01)
syndsize		-0.01***	-0.02***		0.01*	0.00		-0.01***	-0.01***
		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)
lraised		0.01	0.01		0.02***	0.02***		-0.01	-0.00
		(0.00)	(0.00)		(0.01)	(0.01)		(0.00)	(0.00)
cali		0.01	0.01		0.03***	0.03***		-0.00	0.00
		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)
mass		0.06***	0.05***		0.12***	0.10***		0.04***	0.04***
		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)
Industry F.E.	N	N	Y	N	N	Y	N	N	Y
r2	0.005	0.034	0.042	0.017	0.036	0.053	0.005	0.025	0.032
N	111768	107634	107634	32638	30928	30928	79130	76706	76706

I next investigate how changes in underlying asset volatility relate to changes in the VC investment process. Table 8 reports results for regressions on the frequency of reinvestment. The dependent variable is an indicator for whether the same VC firm has a follow-on investment after the round. In general, VC financing relationships are stable in the sense that the same VC firm will usually continue to finance the firm after making an initial investment. The results in Columns (1)-(3) of Table 8 indicate that rising idiosyncratic volatility ($\varepsilon^{CIV}_{t,t+1}$) and market prices ($r^{m}_{t,t+1}$) after the financing round are associated with a greater probability of their being a follow-on financing event from the same VC firm. Columns (4)-(6) and (7)-(8) repeat the analysis in early and late rounds, respectively, where early rounds are those that VentureSource has Identified as being seed, angel or first rounds of investments. In general, early investment rounds are more likely to be involve follow-on investments (65% vs. 52%) and the results in Table 8 imply that these probabilities are also more strongly related to public market conditions. A one-standard-deviation movement in idiosyncratic volatility is associated with a 4% greater probability of follow-on financing for early rounds, while the probability of follow-on financing does not change for late financing rounds. Market returns also have a greater impact on early financing rounds, with a one standard deviation move in market prices associated with a 6% change in the probability of follow-on financing for early rounds and only a 1.3%-1.9% change in later rounds.

Table 9: Time to Next Investment

Statistics are based on OLS regressions using the VentureSource sample of venture capital investments. The dependent variable is the natural logarithm of the number of days from investment to the next investment by the same venture capital firm if such an investment exists. Columns (1)-(3) use the full sample of investments. Columns (4)-(6) use the subsample of early investments. Columns (7)-(9) use the subsample of late investments. Early investments are those that VentureSource designates as seed, angel or first round investments made by a venture capital firm. Standard errors, clustered by quarter, in parentheses: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

	All Rounds			Early Rounds			Late Rounds		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Indus. Q	-0.06***	-0.08***	-0.08***	-0.10***	-0.11***	-0.10***	-0.04**	-0.06***	-0.07***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
CIV_t	-0.03**	-0.02	-0.02	-0.05**	-0.04**	-0.05***	-0.02**	-0.01	-0.01
	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
$\varepsilon^{CIV}_{t,t+1}$	-0.10***	-0.09***	-0.08***	-0.13***	-0.11***	-0.10***	-0.09***	-0.08***	-0.08***
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
$r^m_{t,t+1}$	-0.70***	-0.62***	-0.60***	-0.78***	-0.72***	-0.66***	-0.65***	-0.58***	-0.57***
	(0.11)	(0.11)	(0.11)	(0.15)	(0.14)	(0.13)	(0.10)	(0.11)	(0.11)
supply	-0.02	-0.06***	-0.06***	-0.02	-0.04**	-0.02	-0.02	-0.07***	-0.07***
	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
lage		-0.01	-0.01		0.02***	0.02***		0.04***	0.04***
		(0.01)	(0.01)		(0.00)	(0.00)		(0.01)	(0.01)
syndsize		-0.00	-0.01**		-0.00	-0.01		0.00	-0.00
		(0.00)	(0.00)		(0.01)	(0.01)		(0.00)	(0.00)
lraised		0.17***	0.18***		0.12***	0.12***		0.21***	0.22***
		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)
cali		-0.03**	-0.03**		-0.05***	-0.04***		-0.02	-0.02
		(0.01)	(0.01)		(0.02)	(0.01)		(0.02)	(0.02)
mass		0.00	-0.00		-0.04*	-0.05***		0.02	0.02
		(0.02)	(0.01)		(0.02)	(0.02)		(0.02)	(0.02)
Industry F.E.	N	N	Y	N	N	Y	N	N	Y
r2	0.028	0.104	0.114	0.063	0.104	0.134	0.018	0.128	0.134
N	62411	60724	60724	21322	20621	20621	41089	40103	40103

Table 9 investigates whether, conditional on there being a follow-on financing round, how does the time until that round relate to any changes in underlying asset volatility. The dependent variable, time until the next financing round, is measured as the natural logarithm of the number of days until the next round. The results imply that, conditional on a new financing round occurring, the round is significantly faster in times of rising asset volatility as well as in times of rising market prices.

Finally, I investigate whether realized asset volatility shocks are correlated with returns for individual portfolio company investments. In the theoretical motivation in Section 1.2, the presence of a reinvestment option implies that the returns should depend on *innovations* to idiosyncratic volatility after the company is first financed, and not on the *level* of idiosyncratic volatility at the time of financing. To test this hypothesis, I regress gross cash multiples on levels and innovations of both CIV and market prices:

$$M_{i,t} = \gamma_0 + \gamma_1 CIV_t + \gamma_2 \varepsilon^{CIV}_{t,t+1} + \gamma_3 Q_t + \gamma_4 r^m_{t,t+1} + \delta X + \eta_{i,t}.$$

The variable X is a potential vector of company characteristics and fixed effects.

Table 10: Company Return Dynamics: Cash Multiples

Statistics are based on regressions of the form:

$$M_{i,t} = \gamma_0 + \gamma_1 CIV_t + \gamma_2 \varepsilon^{CIV}_{t,t+1} + \gamma_3 Q_t + \gamma_4 r^m_{t,t+1} + \delta X_{i,t} + \eta_{i,t}$$

where $M_{i,t}$ is the gross cash multiple earned on the investment, CIV_t and Q_t are measured at the date of first financing, $\varepsilon^{CIV}_{t,t+1}$ is the sum of monthly CIV shocks over the year following the first financing and $r^m_{t,t+1}$ is the market return over this same period. The multiple is assumed to be zero if the firm is identified as having defaulted. The variable X is a potential vector of company characteristics and fixed effects. Columns (1)-(3) use the full sample of investments. Columns (4)-(6) use the subsample of early investments. Columns (7)-(9) use the subsample of late investments. Early investments are those that VentureSource designates as seed, angel or first round investments made by a venture capital firm. Standard errors, clustered by quarter, in parentheses: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

	All Rounds			Early Rounds			Late Rounds		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
CIV_t	0.08	-0.00	-0.06	-0.21	-0.11	-0.22	0.13	0.03	-0.00
	(0.12)	(0.12)	(0.12)	(0.33)	(0.33)	(0.32)	(0.11)	(0.11)	(0.11)
$\varepsilon^{CIV}_{t,t+1}$	0.75***	0.60***	0.56***	0.93**	0.82**	0.76*	0.52***	0.43***	0.39***
	(0.13)	(0.13)	(0.13)	(0.31)	(0.31)	(0.30)	(0.11)	(0.11)	(0.12)
Indus. Q	-0.29*	-0.34*	-0.39**	-0.16	-0.07	-0.15	-0.39**	-0.44***	-0.47***
	(0.14)	(0.13)	(0.14)	(0.27)	(0.26)	(0.26)	(0.12)	(0.13)	(0.14)
$r^m_{t,t+1}$	6.26***	5.16***	4.94***	9.70***	8.04***	7.64***	4.01***	3.90***	3.76***
	(0.61)	(0.74)	(0.75)	(1.40)	(1.59)	(1.58)	(0.61)	(0.87)	(0.89)
supply		-0.10	-0.10		-0.33	-0.33		0.06	0.05
		(0.10)	(0.10)		(0.21)	(0.21)		(0.10)	(0.11)
lage		-0.54***	-0.54***		-0.32***	-0.32***		-0.68***	-0.65***
		(0.08)	(0.08)		(0.09)	(0.09)		(0.12)	(0.12)
lraised		-0.25**	-0.25**		-0.49**	-0.51***		-0.01	-0.00
		(0.08)	(0.08)		(0.15)	(0.15)		(0.09)	(0.10)
cali		0.55***	0.49**		1.32***	1.18***		0.15	0.14
		(0.15)	(0.15)		(0.33)	(0.32)		(0.16)	(0.17)
mass		0.18	0.12		1.04*	0.97		-0.20	-0.23
		(0.19)	(0.20)		(0.51)	(0.50)		(0.19)	(0.20)
Industry F.E.	N	N	Y	N	N	Y	N	N	Y
r2	0.045	0.065	0.070	0.059	0.076	0.084	0.040	0.051	0.057
N	20780	20780	20780	6416	6416	6416	14364	14364	14364

Results for this regression are in Table 10. Starting with column (1), which does not include firm-level controls or fixed effects, we can see that the coefficient on the level of CIV at first financing enters insignificantly, consistent with the theoretical motivation. The level of asset volatility appears to be priced into portfolio company investments, consistent with competitive financing markets. The innovation to CIV ($\varepsilon_{t,t+1}^{CIV}$), on the other hand, enters positively and significantly. Market prices enter similarly, with the coefficient Tobin's Q entering with only marginal significance and market returns after the investment strongly statistically significant. Columns (2) and (3) include company-level controls and industry fixed effects, neither of which materially impact these results dramatically. Columns (4)-(6) and (7)-(9) repeat the analysis in early and late rounds, respectively. Similar to the results reported in Sections 1.4.2 and 1.4.3, the return of early-stage investments are much more strongly related to innovations in asset volatility than those for late investment rounds.

1.5. Conclusion

The contractual arrangements through which VC firms invest in their portfolio companies, in particular the real option embedded in staged financing, lead these funds to be exposed to changes in the idiosyncratic volatility of these companies. The idiosyncratic return volatility of these companies is, in turn, exposed to aggregate levels of idiosyncratic volatility through a common factor structure. I find that this channel explains a significant fraction of the historical performance of VC investments.

At the asset class level, VC indexes from Cambridge Associates and Sand Hill Econometrics show significant exposure to idiosyncratic volatility shocks, with a one standard deviation quarterly shock leading to an 8% return in the CA index. This loading is even larger for a benchmark index of early stage investment focused funds and is smaller for balanced- and late-focused funds. An index of buyout funds shows no exposure to idiosyncratic volatility shocks. I repeat the analysis using a tradeable proxy for idiosyncratic volatility shocks constructed from equity options and find that the exposure to this proxy increases risk-adjusted excess returns. One possible interpretation of this result is that investors who ignore this valuable risk exposure undervalue the returns to venture capital investments.

At the investment level, I find that positive innovations to idiosyncratic volatility that occur during the fund's life lead to greater fund performance relative to a market benchmark, and that the shocks with the greatest impact are those that occur in years three and four of the fund's life. Again, funds that focus on early-stage investments, i.e. those with more reinvestment options, experience much greater performance exposure to idiosyncratic volatility shocks.

At the level of individual portfolio company investments, I find that the average cash multiple is greater for those investments that experience positive innovations to idiosyncratic volatility early in their lives, but that the level of idiosyncratic volatility does not predict returns, consistent with rational and competitive pricing in the market for entrepreneurial finance.

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CHAPTER 2 : Intangible Capital and the Investment-q Relation

2.1. Introduction

The neoclassical theory of investment was developed more than 30 years ago, when firms mainly owned physical assets such as property, plant, and equipment (PP&E). As a result, empirical tests of the theory have focused almost exclusively on physical capital. Since then, the US economy has shifted toward service- and technology-based industries, which has made intangible assets such as human capital, innovative products, brands, patents, software, customer relationships, databases, and distribution systems increasingly important. Corrado and Hulten (2010) estimate that intangible capital makes up 34% of firms' total capital in recent years. Despite the importance of intangible capital, researchers have almost always excluded it when testing investment theories.

Is there a role for intangible capital in the neoclassical theory of investment? If so, how must empirical tests be adapted? Is the theory still relevant in an economy increasingly dominated by intangible capital? For example, the Hayashi (1982) classic q-theory of investment predicts that Tobin's q, the ratio of capital's market value to its replacement cost, perfectly summarizes a firm's investment opportunities. As a result, Tobin's q has become "arguably the most common regressor in corporate finance" (Erickson and Whited, 2012, p. 1286). How should researchers proxy for investment opportunities in an increasingly intangible economy? And how well do those proxies work?

To answer these questions, we revisit the basic empirical facts about the relation between corporate investment, Tobin's q, and free cash flow. A very large investment literature, both in corporate finance and macroeconomics, is built upon these fundamental facts, so it is important to understand how the facts change when accounting for intangible capital. We show that some facts do change significantly, and we discuss the implications for our theories of investment. Most important, we show that the classic q theory of investment, despite originally being designed to explain physical investment, also helps explain intangible investment. In other words, the neoclassical theory of investment is still relevant. An important component of our analysis is a new Tobin's q proxy that accounts for intangible capital. We show that this new proxy captures firms' investment opportunities better than other popular proxies, thus offering a simple way to improve corporate finance regressions without additional econometrics.

To guide our empirical work, we begin with a theory of a firm that invests optimally in physical and intangible capital over time. The theory is a standard neoclassical investment-q theory in the spirit of Hayashi (1982) and Abel and Eberly (1994). Like physical capital, intangible capital is costly to obtain and helps produce future profits, albeit with some risk. For this fundamental reason, it makes sense to treat intangible capital as capital in the neoclassical framework. Our theory predicts that a firm's physical and intangible investment rates should both be explained well by a version of Tobin's q that we call "total q," which equals the firm's market value divided by the sum of its physical and intangible capital stocks.

We test this and other predictions using data on public US firms from 1975 to 2011. We measure a firm's intangible capital as the sum of its knowledge capital and organization capital. We interpret research and development (R&D) spending as an investment in knowledge capital, and we apply the perpetual-inventory method to a firm's past R&D to measure the replacement cost of its knowledge capital. We similarly interpret a fraction of past selling, general, and administrative (SG&A) spending as an investment in organization capital, which includes human capital, brand, customer relationships, and distribution systems. Our measure of intangible capital builds on the measures of Lev and Radhakrishnan (2005), Corrado, Hulten, and Sichel (2009), Corrado and Hulten (2010, 2014), Eisfeldt and Papanikolaou (2013, 2014), Falato, Kadyrzhanova, and Sim (2013), and Zhang (2014). We define a firm's total capital as the sum of its physical and intangible capital, both measured at replacement cost. Guided by our theory, we measure total q as the firm's market value divided by its total capital, and we scale the physical and intangible investment rates by total capital.

While our intangible-capital measure has limitations, we believe, and the data confirm, that an imperfect proxy is better than setting intangible capital to zero. A benefit of the measure is that it is easily computed for all public US firms back to 1975, and it requires only Compustat data and other easily downloaded data. Our data on firms' total q and intangible capital can be downloaded from Wharton Research Data Services (WRDS).

Our analysis begins with ordinary least squares (OLS) panel regressions of investment on q. Consistent with our theory, total q explains physical and intangible investment roughly equally well. Their within-firm R^2 values are 21% and 28%, respectively. Total q explains the sum of physical and intangible investment (total investment) even better, delivering an R^2 of 33%. Judging by R^2, the neoclassical theory of investment works at least as well for intangible capital as for physical capital, and it works even better for an all-inclusive measure of capital. Also consistent with our theory, the literature's standard investment regression, which excludes intangible capital, typically delivers lower R^2 values.

According to the theory, physical and intangible investment should co-move, because they share the same marginal productivity of capital, as proxied by total q. The data support this view: The within-firm correlation between physical and intangible investment is 31% but drops to 17% after controlling for total q.

Throughout the corporate finance literature, researchers use Tobin's q to proxy for firms' investment opportunities. Our OLS R^2 values help evaluate these proxies. We find that including intangible capital in our q measure produces a superior proxy for investment opportunities, no matter how we measure investment. We compare total q with the investment literature's standard q measure, which scales firm value by physical capital (PP&E) alone. Total q is better at explaining physical, intangible, and total investment, as well as R&D investment and the literature's standard investment measure, capital expenditure (CAPX) scaled by PP&E. It is also popular to measure Tobin's q as the firm's market value scaled by the book value of assets. The problem with this measure is that "Assets" on the balance sheet excludes the vast majority of firms' intangible capital, because US accounting rules treat R&D and SG&A as operating expenses, not capital investments. Like Erickson and Whited (2006, 2012), we find that market-to-book–assets ratios are especially poor proxies for investment opportunities.

The OLS regressions suffer from two well-known problems. First, the slopes on q are biased due to measurement error in q. Second, the OLS R^2 depends not just on how well q explains investment, but also on how well our q proxies explain the true, unobservable q. To address these problems, we reestimate the investment models using the Erickson, Jiang, and Whited (2014) cumulant estimator. This estimator produces unbiased slopes and a statistic τ^2 that measures how close our q proxy is to the true, unobservable q. Specifically, τ^2 is the R^2 from a hypothetical regression of our q proxy on the true q. We find that τ^2 is 21% higher when we include intangible capital in the investment-q regression, implying that our new q proxy is closer to the true q.

According to our theory, slope coefficients of investment on total q help measure capital adjustment costs. The inverse q-slope for physical (intangible) investment measures the convex component of physical (intangible) capital's adjustment costs. We find that intangible investment's q-slope is roughly half as large as physical investment's, implying intangible capital's convex adjustment costs are twice as large as those for physical capital. This finding supports the literature's conjecture that intangible capital is costlier than physical capital to adjust, because adjusting intangible capital often requires replacing highly trained employees (e.g., Grabowski, 1968; Brown, Fazzari, and Petersen, 2009). An important implication of our result is that firms adjust more slowly to changes in investment opportunities as the economy shifts toward intangible capital. We also find that accounting for intangibles roughly doubles the q-slope for physical investment, implying significantly lower convex adjustment costs for physical capital than previously believed.

Like other simple q theories, ours predicts that cash flow should not help explain investment after controlling for q. Researchers typically measure cash flow as profits net of R&D and SG&A. Because R&D and at least part of SG&A are actually investments, one should add them back to measure cash flow available for investment. After making this adjustment, we find that physical investment becomes more sensitive to cash flow than previously believed. On this dimension, the neoclassical theory fits the data worse after accounting for intangibles. In contrast, the R&D component of intangible investment is insensitive to cash flow, supporting the theory. Because SG&A's investment component is difficult to measure, it remains unclear whether intangible investment overall is more sensitive than physical investment to cash flow. Financing constraints are unlikely to explain the opposing cash flow results for physical and R&D capital, as financing constraints are arguably more severe for R&D capital due to its lower collateral value (Almeida and Campello, 2007; Falato, Kadyrzhanova, and Sim, 2013). More recent theories predict an investment–cash flow sensitivity even without financing constraints.[1] For example, diseconomies of scale can make cash flow informative about investment opportunities, even controlling for Tobin's q. Without a full structural estimation, it is difficult to tell whether our cash flow results are driven by differences in financing constraints, diseconomies of scale, or some other source.

Several important investment studies use data only from manufacturing firms.[2] Surprisingly, we find that the classic q theory fits the data better outside the manufacturing industry and, more generally, in firms and years with more intangible capital. Investment is usually better explained by q and is less sensitive to cash flow in subsamples with more intangibles. These results even hold using the literature's standard measures that exclude intangibles. Again, our results imply that the neoclassical theory of investment is just as relevant, if not more so, in an increasingly intangible economy. Why the theory fits better in high-intangible settings remains unclear. We find no robust evidence that high-intangible firms

[1] Examples include Gomes (2001), Alti (2003), Cooper and Ejarque (2003), Hennessy and Whited (2007), Abel and Eberly (2011), Gourio and Rudanko (2014), and Abel (2016).

[2] Examples include Fazzari, Hubbard, and Petersen (1988), Almeida and Campello (2007), and Erickson and Whited (2012). A common reason is that manufacturing firms' capital is easier to measure. Our τ^2 statistics confirm that the literature's standard q proxy has less measurement error in the manufacturing industry compared with other industries.

are closer to the theory's ideal of perfect competition and constant returns to scale. Also, high-intangible firms arguably face more financing constraints, which should make theory fit worse, not better.

Some of our main results are even stronger in macroeconomic time series data. For example, the literature's standard investment-q regression, which excludes intangibles, delivers an R^2 of just 4%, whereas the regression including intangible capital produces an R^2 of 61%. In first differences, total q explains physical and intangible capital roughly equally well. Again, the neoclassical theory of investment applies just as well, if not better, to intangible capital.

The empirical investment-q literature is extensive and dates back at least to Ciccolo (1975) and Abel (1980). Hassett and Hubbard (1997) and Caballero (1999) review the literature. Tests of the classic q theory using physical capital have been disappointing. Investment is typically sensitive to cash flow, is explained poorly by q (low R^2), and produces implausibly large adjustment-cost parameters (low q-slopes). We show that including intangible capital helps solve the last two problems but not the first one. Other attempts to solve these problems with better measurement include using a fundamental q instead of market values directly (Abel and Blanchard, 1986), using bond prices (Philippon, 2009), correcting for measurement error (Erickson and Whited, 2000, 2012; Erickson, Jiang, and Whited, 2014), and using state variables directly (Gala and Gomes, 2013). We also correct for measurement error, and we show that including intangibles yields even larger improvements than using bond prices.

Our paper is not the first to examine the empirical relation between intangible investment and Tobin's q. Eisfeldt and Papanikolaou (2013) find a positive relation between investment in organization capital and q. Almeida and Campello (2007) and others use q and cash flow to forecast R&D investment. Chen, Goldstein, and Jiang (2007) use q to forecast the sum of physical investment and R&D. Closer to our specifications, Baker, Stein, and Wurgler (2002) measure investment as the sum of CAPX, R&D, and SG&A, and they regress them on q. Gourio and Rudanko (2014) examine the relation between q and investment in customer capital, a type of intangible capital. All these papers use a q proxy that excludes intangibles from the denominator. Besides having a different focus, our paper is the first to include all types of intangible capital not just in investment, but also in Tobin's q and cash flow. Including intangibles in all three measures is important for delivering our results. Belo, Lin, and Vitorino (2014) show that physical and brand investment are both procyclical, which is related to our co-movement result, but they do not examine Tobin's q.

Almeida and Campello (2007) examine how asset tangibility and financial constraints affect the investment–cash flow relation. Like us, they find a higher investment–cash flow sensitivity for firms using less intangibles. Unlike our measures of asset intangibility, theirs exclude firms' internally created intangible assets, which we find make up the vast majority of intangible capital.

Li, Liu, and Xue (2014) structurally estimate a q-theory model that includes intangible capital. Like us, they find that intangible capital has larger adjustment-cost parameters than physical capital and that including intangibles decreases physical capital's estimated adjustment costs. Unlike us, they focus on the cross section of stock returns, and they exclude organization capital.

The paper proceeds as follows. Section 2.2 presents our theory of investment in physical and intangible capital. Section 2.3 describes the data and intangible-capital measure we use to test the theory's predictions. Section 2.4 presents full-sample results, and Section 2.5 compares results across different types of firms, industries, and years. Section 2.6 contains results for the overall macroeconomy. Section 2.7 explores the robustness of our empirical results, and Section 2.8 concludes.

2.2. Intangible capital and the neoclassical theory of investment

In this section, we review the neoclassical theory of investment, and we argue that intangible capital fits well into the theory. We simplify and modify the Abel and Eberly (1994) theory of investment under uncertainty to include two capital goods that we interpret as physical and intangible capital. We present a stylized model, as our goal is to provide theoretical motivation for our empirical work, not to make a theoretical contribution. Wildasin (1984), Hayashi and Inoue (1991), and others already provide theories of investment in multiple capital goods. We first present the model's assumptions and predictions, and then we discuss them.

2.2.1. Model assumptions and empirical implications

The model features an infinitely lived, perfectly competitive firm i that holds K_{it}^{phy} units of physical capital and K_{it}^{int} units of intangible capital at time t. The firm's total capital is defined as $K^{tot} = K^{phy} + K^{int}$. At each instant t, the firm chooses the investment rates I^{phy} and I^{int} that maximize firm value V_{it}:

$$V_{it} = \max_{I_{i,t+s}^{phy},\ I_{i,t+s}^{int}} \int_0^{\infty} E_t[\Pi\left(K_{i,t+s}^{tot}, \varepsilon_{i,t+s}\right) - c_i^{phy}\left(I_{i,t+s}^{phy}, K_{i,t+s}^{tot}, p_{i,t+s}^{phy}\right)$$

$$-c_i^{int}\left(I_{i,t+s}^{int}, K_{i,t+s}^{tot}, p_{i,t+s}^{int}\right)]e^{-rs}ds, \tag{2.1}$$

subject to

$$dK^m = \left(I^m - \delta K^m\right) dt,\ m = phy,\ int. \tag{2.2}$$

Both types of capital depreciate at the same rate δ. The profit function Π depends on a shock ε and is assumed linearly homogenous in K^{tot}. The two investment-cost functions c equal

$$c_i^m\left(I^m, K^{tot}, p^m\right) = p^m I^m + K^{tot}\left[\zeta_i^m \frac{I^m}{K^{tot}} + \frac{\gamma_i^m}{2}\left(\frac{I^m}{K^{tot}}\right)^2\right],\ m = phy,\ int, \tag{2.3}$$

where $\gamma_i > 0$. The first term denotes the direct purchase or sale cost of investment, with each new unit of capital costing p^m. The second term equals the cost of adjusting the stock of capital type m. Capital prices p_{it}^{phy} and p_{it}^{int}, along with profitability shock ε_{it}, fluctuate over time according to a general stochastic diffusion process:

$$dy_{it} = \mu\left(y_{it}\right)dt + \Sigma\left(y_{it}\right)dB_{it}, \tag{2.4}$$

where $y_{it} = \left[\varepsilon_{it} \;\; p_{it}^{phy} \;\; p_{it}^{int}\right]'$.

We have four main predictions. All proofs are in Appendix A.

Prediction 1. Physical and intangible capital share the same marginal q. Marginal q equals average q, the ratio of firm value to its total capital stock:

$$\frac{\partial V_{it}}{\partial K_{it}^{phy}} = \frac{\partial V_{it}}{\partial K_{it}^{int}} = \frac{\partial V_{it}}{\partial K_{it}^{tot}} = \frac{V_{it}}{K_{it}^{tot}} \equiv q^{tot}\left(\varepsilon_{it}, p_{it}^{phy}, p_{it}^{int}\right). \tag{2.5}$$

Marginal q equals $\partial V/\partial K$ and measures the benefit of adding an incremental unit of capital (either physical or intangible) to the firm. Marginal q equals average q, because we assume constant returns to scale, perfect competition, and perfect substitutes in production and depreciation. This prediction provides a rationale for measuring Tobin's q as q^{tot}, firm value divided by K^{tot}, the sum of physical and intangible capital. The value of q^{tot} depends endogenously on the shock ε and the two capital prices.

The firm chooses its optimal investment rates by equating their marginal q and their marginal cost of investment. Applying this condition to Eq. (2.3) yields Prediction 2.

Prediction 2. The firm's optimal physical and intangible investment rates follow

$$\iota_{it}^{phy} = \frac{I_{it}^{phy}}{K_{it}^{tot}} = \frac{1}{\gamma_i^{phy}}\left(q_{it}^{tot} - \zeta_i^{phy} - p_{it}^{phy}\right) \tag{2.6}$$

$$\iota_{it}^{int} = \frac{I_{it}^{int}}{K_{it}^{tot}} = \frac{1}{\gamma_i^{int}}\left(q_{it}^{tot} - \zeta_i^{int} - p_{it}^{int}\right). \tag{2.7}$$

Prediction 2 says that the physical and intangible investment rates, both scaled by total capital, vary with q^{tot}. One empirical implication is that physical and intangible investment rates should be correlated. The correlation might not be perfect, though, because adjustment-cost parameters might not be perfectly correlated across firms, and the prices p_{it}^{phy} and p_{it}^{int} might not be perfectly correlated either across firms or over time.

Predictions 3 and 4 follow immediately from Prediction 2 and form the basis of our empirical work. Consider a panel of firms indexed by i. We assume parameters γ^{phy} and γ^{int} are constant across firms, but other parameters and shocks can vary across firms. We assume the two capital prices p_{it}^{m} can be decomposed as $p_i^m + p_t^m$.

Prediction 3. In an OLS panel regression of ι_{it}^{phy} on q_{it}^{tot} and firm and time fixed effects (FEs), the slope on q equals $1/\gamma^{phy}$. If the dependent variable is instead ι_{it}^{int}, the $q-$slope equals $1/\gamma^{int}$. If the dependent variable is ι_{it}^{tot}, the $q-$slope equals $1/\gamma^{phy} + 1/\gamma^{int}$. Any other regressors, such as free cash flow, should not enter significantly if added to any of these regressions.

Prediction 3 says that total q helps explain all three investment measures, and it shows that the OLS slopes identify the adjustment-cost parameters γ. The firm and year FEs are needed to absorb the terms $-\zeta_i - p_{it}$ in Eqs. (2.6) and (2.7).

To our knowledge, Prediction 4 is new to the literature. It helps explain the investment literature's typical regression, which excludes intangible capital and instead scales investment and q by physical capital alone.

Prediction 4. Define $q_{it}^* = V_{it}/K_{it}^{phy}$ and $\iota_{it}^* = I_{it}^{phy}/K_{it}^{phy}$. In an OLS panel regression of ι_{it}^* on q_{it}^* and firm and time fixed effects, the slope coefficient is a downward-biased estimate of $1/\gamma^{phy}$, and the R^2 is lower than the R^2 from the regressions in Prediction 3.

According to our theory, this regression is misspecified, because the ratio $-K_{it}^{tot}/K_{it}^{phy}$ is part of the regression's disturbance and cannot be explained by the FEs. Its q-slope is downward-biased, meaning it produces upward-biased estimates of the adjustment-cost parameter γ^{phy}, because q_{it}^* depends on the ratio $K_{it}^{tot}/K_{it}^{phy}$, making the regressor negatively related to the disturbance.

At this point, we have imposed several restrictive assumptions. To help judge the model's empirical relevance, we establish one last prediction and use it as a consistency check in our empirical work. This last prediction links firms' use of intangible capital to their adjustment costs and q-slopes. If we impose the additional assumptions that physical and intangible capital have the same linear adjustment-cost parameters ($\zeta_i^{phy} = \zeta_i^{int}$) and purchase prices ($p_{it}^{phy} = p_{it}^{int}$), then

$$\lim_{t \to \infty} \frac{K_{it}^{int}}{K_{it}^{tot}} = \frac{\gamma^{phy}}{\gamma^{phy} + \gamma^{int}} = \frac{\beta^{int}}{\beta^{int} + \beta^{phy}}, \tag{2.8}$$

where β^{int} and β^{phy} are the Prediction 3 slopes of ι^{int} and ι^{phy}, respectively, on q^{tot}. Intuitively, if physical and intangible capital are identical except for their adjustment-cost parameters γ, then a firm holds relatively less intangible capital if intangible capital is costlier to adjust ($\gamma^{int} > \gamma^{phy}$). Section 5 performs a consistency check by comparing the Eq. (2.8) ratio of regression slopes across firms with different amounts of intangible capital.

2.2.2. Discussion

To summarize, our simple theory predicts that total q helps explains physical, intangible, and total investments when we scale them by the firm's total capital. It also illustrates how investment regressions can identify the convex part (γ) of capital adjustment costs. The theory also predicts that including intangible capital produces a better-specified investment regression and more accurate adjustment-cost estimates.

Next, we discuss the theory's assumptions and limitations. Overall, we argue that intangible capital fits well into the neoclassical framework.

Conceptually, spending on intangible assets qualifies as a capital investment, because it reduces current cash flow to increase future cash flow (Corrado, Hulten, and Sichel, 2005, 2009). Ample evidence exists that intangible investments increase firms' future profits, as our theory assumes. A large R&D literature (e.g., Lev and Sougiannis, 1996) shows that R&D investments increase firms' future profits. Recognizing this fact, the Bureau of Economic Analysis (BEA) began capitalizing R&D in satellite accounts in 1994 and in core National Income and Product Accounts (NIPA) in 2013. A large marketing literature (e.g., Aaker, 1991; Srivastava, Shervani, and Fahey, 1997) shows that firms with stronger brands earn higher profits and are worth more. More generally, Eisfeldt and Papanikoloau (2013) show that firms using more organization capital are more productive after accounting for physical capital and labor. Even though a firm does not own its workers, employee training builds the firm's human capital, because training is costly and increases the firm's future profits.

While employee training and brand building can entail relatively low risk, investments such as R&D projects are highly risky and sometimes fail completely. The same is true for physical investments, though. Our theory is designed to handle investments with risky payoffs, so payoff risk is no reason to exclude intangible capital from the neoclassical theory. In addition to payoff risk, firms face depreciation risk. Our theory assumes a constant depreciation rate for intangible capital, whereas the true rate is likely random. For example, writing off a large portion of knowledge capital could be appropriate when a firm narrowly loses a patent race. Physical capital's true depreciation rate is also likely random, however. For example, an unexpected product-market change could make a machine obsolete. Again, no conceptual difference exists between physical and intangible capital here, although there could be a difference of degree.

When researchers test investment theories, they usually measure investment as CAPX and capital as PP&E. These two measures add together physical assets that are conceptually very different from each other, such as timberland, medical equipment, oil reserves, computers, buildings, and so on. By using such measures, researchers implicitly treat these physical assets as perfect substitutes. Similarly, our theory adds together many different types of intangible assets into K^{int}, and then it assumes the firm's profits depend on K^{tot}, the sum of physical and intangible capital. We therefore treat all assets as perfect substitutes in producing profits, although we do allow them to have potentially different adjustment costs. In our opinion, a natural first step is to treat intangible capital the same way researchers for decades have treated physical capital. In reality, physical and intangible capital could be complements, not substitutes. One might therefore expect our empirical measures, which simply add together all capital, to produce poor results. We find the opposite, which is somewhat surprising and suggests that our simple model provides a useful approximation of reality.

The theory highlights an important limitation of investment regressions. Whited (1994) and Erickson and Whited (2000) explain that investment regressions cannot identify the level of adjustment costs. For example, our theory predicts that the linear adjustment-cost parameters ζ are not separately identified from firm-specific capital prices p. The investment regression identifies only the quadratic adjustment-cost parameters γ, meaning the investment regression can identify only the convex component of adjustment costs. This convex component is interesting, however, as it determines how investment responds to investment opportunities.

2.3. Firm-level data

Our sample includes all Compustat firms except regulated utilities (Standard Industrial Classification codes 4900–4999), financial firms (6000–6999), and firms categorized as public service, international affairs, or non-operating establishments (9000+). We also exclude firms with missing or non-positive book value of assets or sales and firms with less that \$5 million in physical capital, as is standard in the literature. We use data from 1975 to 2011, although we use earlier data to estimate firms' intangible capital. Our sample starts in 1975, because this is the first year that the Federal Accounting Standards Board (FASB) requires firms to report R&D. We winsorize all regression variables at the 1% level to remove outliers.

2.3.1. Tobin's q

Guided by our theory, we measure total q by scaling firm value by the sum of physical and intangible capital:

$$q_{it}^{tot} = \frac{V_{it}}{K_{it}^{phy} + K_{it}^{int}}. \tag{2.9}$$

We measure the replacement cost of physical capital, K^{phy}, as the book value of property, plant, and equipment (Compustat item *ppegt*). Subsection 3.2 defines our measure of K^{int}, the replacement cost of intangible capital. We measure the firm's market value V as the market value of outstanding equity (Compustat items *prcc_f* times *csho*), plus the book value of debt (Compustat items $dltt + dlc$), minus the firm's current assets (Compustat item *act*), which include cash, inventory, and marketable securities.

For comparison, we examine the literature's standard Tobin's q measure used by Fazzari, Hubbard, and Petersen (1988), Erickson and Whited (2012), and many others:

$$q_{it}^{*} = \frac{V_{it}}{K_{it}^{phy}}. \tag{2.10}$$

Erickson and Whited (2006, 2012) compare several alternate Tobin's q measures, including the market-to-book–assets ratio, and they find that q^* best explains investment. The correlation between q^* and q^{tot} is 0.82.

2.3.2. Intangible capital

We briefly review the US accounting rules for intangible capital before defining our measure.[3] The accounting rules depend on whether the firm creates the intangible asset internally or purchases it externally.

Intangible assets created within a firm are expensed on the income statement and almost never appear as assets on the balance sheet. For example, a firm's spending to develop

[3] Chapter 12 in Kieso, Weygandt, and Warfield (2010) provides a useful summary of the accounting rules for intangible assets. The authors also provide references to relevant FASB codifications.

knowledge, patents, or software is expensed as R&D. Advertising to build brand capital is a selling expense within SG&A. Employee training to build human capital is a general or administrative expense within SG&A. There are a few exceptions, in which internally created intangibles are capitalized on the balance sheet, but these are small in magnitude.[4]

When a firm purchases an intangible asset externally, for example, by acquiring another firm, the firm typically capitalizes the asset on the balance sheet as part of Intangible Assets, which equals the sum of Goodwill and Other Intangible Assets. The asset is booked in Other Intangible Assets if the acquired asset is separately identifiable, such as a patent, software, or client list. Acquired assets that are not separately identifiable, such as human capital, are in Goodwill. When an intangible asset becomes impaired, firms are required to write down its book value.

We define the replacement cost of intangible capital, denoted K^{int}, to be the sum of the firm's externally purchased and internally created intangible capital. We measure externally purchased intangible capital as Intangible Assets from the balance sheet (Compustat item *intan*). We set this value to zero if missing. We keep Goodwill in Intangible Assets in our main analysis, because Goodwill does include the fair cost of acquiring intangible assets that are not separately identifiable. Because Goodwill can be contaminated by non-intangibles, such as a market premium for physical assets, we also try excluding Goodwill from external intangibles and show that our results are almost unchanged (Section 7). Our mean (median) firm purchases only 19% (3%) of its intangible capital externally, meaning the vast majority of firms' intangible assets are missing from their balance sheets. There are important outliers, however. For example, 41% of Google's intangible capital in 2013 had been purchased externally. Including these externally purchased intangibles is an innovation in our measure relative to those in the literature.

[4]Our measure captures these exceptions via balance sheet Intangible Assets. Firms capitalize the legal costs, consulting fees, and registration fees incurred when developing a patent or trademark. A firm can start capitalizing software spending only after the product reaches technological feasibility (for externally sold software) or reaches the coding phase (for internally used software). The resulting software asset is part of Other Intangible Assets (*intano*) in Compustat.

Measuring the replacement cost of internally created intangible assets is difficult, as they appear nowhere on the balance sheet. Fortunately, we can construct a proxy by accumulating past intangible investments, as reported on firms' income statements. We define the stock of internal intangible capital as the sum of knowledge capital and organization capital.

A firm develops knowledge capital by spending on R&D. We estimate a firm's knowledge capital by accumulating past R&D spending using the perpetual inventory method:

$$G_{it} = (1 - \delta_{R\&D})G_{i,t-1} + R\&D_{it}, \tag{2.11}$$

where G_{it} is the end-of-period stock of knowledge capital, $\delta_{R\&D}$ is its depreciation rate, and $R\&D_{it}$ is real expenditures on R&D during the year. The BEA uses a similar method to capitalize R&D, as do practitioners when valuing companies (Damodaran, 1999, 2001). For $\delta_{R\&D}$, we use the BEA's industry-specific R&D depreciation rates.[5] We measure annual R&D using the Compustat variable *xrd*. We use Compustat data back to 1950 to compute Eq. (2.11), but our regressions include only observations starting in 1975. Starting in 1977, we set R&D to zero when missing, following Lev and Radhakrishnan (2005) and others.[6]

One challenge in applying the perpetual inventory method in Eq. (2.11) is choosing a value for G_{i0}, the capital stock in the firm's first non-missing Compustat record, which usually coincides with the initial public offering (IPO). We estimate G_{i0} using data on the firm's founding year, R&D spending in its first Compustat record, and average pre-IPO R&D growth rates. With these data, we estimate the firm's R&D spending in each year between its founding and appearance in Compustat. We apply a similar approach to

[5] The BEA's R&D depreciation rates are from the analysis of Li (2012). The depreciation rates range from 10% in the pharmaceutical industry to 40% for computers and peripheral equipment. Following the BEA's guidance, we use a depreciation rate of 15% for industries not in Li's Table 4. Our results are virtually unchanged if we set $\delta_{R\&D}$ equal to 10%, 15%, or 20% for all industries (Table 19).

[6] We start in 1977 to give firms two years to comply with FASB's 1975 R&D reporting requirement. If we see a firm with R&D equal to zero or missing in 1977, we assume the firm was typically not an R&D spender before 1977, so we set any missing R&D values before 1977 to zero. Otherwise, before 1977, we either interpolate between the most recent non-missing R&D values (if such observations exist) or we use the method in Appendix A (if those observations do not exist). Starting in 1977, we make exceptions in cases in which the firm's assets are also missing. These are likely years when the firm was privately owned. In such cases, we interpolate R&D values using the nearest non-missing values.

SG&A. Appendix B provides additional details. Section 2.7 shows that a simpler measure assuming $G_{i0} = 0$ produces an even stronger investment-q relation than our main measure. We consider that simpler measure a reasonable alternate proxy for investment opportunities.

Next, we measure the stock of organization capital by accumulating a fraction of past SG&A spending using the perpetual inventory method, as in Eq. (2.11). The logic is that at least part of SG&A represents an investment in organization capital through advertising, spending on distribution systems, employee training, and payments to strategy consultants. We follow Hulten and Hao (2008), Eisfeldt and Papanikoloau (2014), and Zhang (2014) in counting only 30% of SG&A spending as an investment in intangible capital. We interpret the remaining 70% as operating costs that support the current period's profits. Section 2.7 shows that our conclusions still go through, albeit with smaller magnitudes, if we use values other than 30%. We follow Falato, Kadyrzhanova, and Sim (2013) in using a depreciation rate of $\delta_{SG\&A} = 20\%$, and in Section 2.7 we show that our conclusions are robust to alternate depreciation rates.

Measuring SG&A from Compustat data is not trivial. Companies typically report SG&A and R&D separately. Compustat, however, almost always adds them together in a variable misleadingly labeled "Selling, General and Administrative Expense" (item $xsga$). We must therefore subtract xrd from $xsga$ to isolate the SG&A that companies report. Appendix B provides additional details.

Our measure of internally created organization capital is almost identical to that of Eisfeldt and Papanikolaou (2012, 2013, 2014). They validate the measure in several ways. They show a positive correlation between firms' use of organization capital and the Bloom and Van Reenen (2007) managerial quality score. This score is associated with higher firm profitability, production efficiency, and productivity of information technology (IT) (Bloom, Sadun, and Van Reenen, 2010). Eisfeldt and Papanikoloau (2013) show that firms using more organization capital are more productive after accounting for physical capital and labor, spend more on IT, and employ higher-skilled workers. They show that firms with more organization capital list the loss of key personnel as a risk factor more often in their 10-K filings. Practitioners also use our approach. A popular textbook on value investing recommends capitalizing SG&A to measure assets missing from the balance sheet (Greenwald, Kahn, Sonkin, and Van Biema, 2004).

Our measure of intangible capital has the benefit of being easily computed for the full Compustat sample. The measure has limitations, however, as discussed in Subsection 2.2. Subsection 2.4.2 addresses concerns about measurement error bias, and Section 2.7 shows that our conclusions are robust to several alternate ways of measuring intangible capital. Overall, we believe, and the data confirm, that an imperfect proxy for intangible capital is better than setting it to zero.

2.3.3. Investment

Guided by our theory, we measure the firm's physical, intangible, and total investment rates as

$$\iota_{it}^{phy} = \frac{I_{it}^{phy}}{K_{i,t-1}^{tot}}, \qquad \iota_{it}^{int} = \frac{I_{it}^{int}}{K_{i,t-1}^{tot}}, \qquad \iota_{it}^{tot} = \iota_{it}^{phy} + \iota_{it}^{int}. \tag{2.12}$$

We measure physical investment I^{phy} as capital expenditures (Compustat item *capx*), and we measure intangible investment, I^{int}, as R&D + (0.3 $\times$ SG&A). This definition assumes 30% of SG&A represents an investment, as we assume when estimating capital stocks. For comparison, we examine the literature's standard physical investment measure, denoted ι^* in our theory:

$$\iota_{it}^{*} = \frac{I_{it}^{phy}}{K_{i,t-1}^{phy}}. \tag{2.13}$$

The correlation between ι^{phy} and ι^* is 0.83.

2.3.4. *Cash flow*

Erickson and Whited (2012), Almeida and Campello (2007), and others measure free cash flow as

$$c^*_{it} = \frac{IB_{it} + DP_{it}}{K^{phy}_{i,t-1}}, \tag{2.14}$$

where IB is income before extraordinary items and DP is depreciation expense. The measure c^* is the pre-depreciation free cash flow available for physical investment or distribution to shareholders. One shortcoming of c^* is that it treats R&D and SG&A as operating expenses, not investments. In addition to the standard measure c^*, we use an alternate cash flow measure that recognizes R&D and part of SG&A as investments. We add intangible investments back into the free cash flow so that we measure the profits available for total, not just physical, investment:

$$c^{tot}_{it} = \frac{IB_{it} + DP_{it} + I^{int}_{it}(1-\kappa)}{K^{phy}_{i,t-1} + K^{int}_{i,t-1}}. \tag{2.15}$$

Lev and Sougiannis (1996) similarly adjust earnings for intangible investments, as do practitioners (Damodaran, 1999, 2001). Because accounting rules allow firms to expense intangible investments, the effective cost of a dollar of intangible capital is only $(1-\kappa)$, where κ is the marginal tax rate. When available, we use simulated marginal tax rates from Graham (1996). Otherwise, we assume a marginal tax rate of 30%, which is close to the mean tax rate in the sample. The correlation between c^{tot} and c^* is 0.77.

2.3.5. *Summary statistics*

Table **??** contains summary statistics. We define intangible intensity as a firm's ratio of intangible to total capital, at replacement cost. The mean (median) intangible intensity is 43% (45%), so almost half of capital is intangible in our typical firm-year. Knowledge capital makes up only 24% of intangible capital on average, so organization capital makes up 76%. The median firm has almost no knowledge capital, as almost half of firms report no

Table 11: Summary Statistics

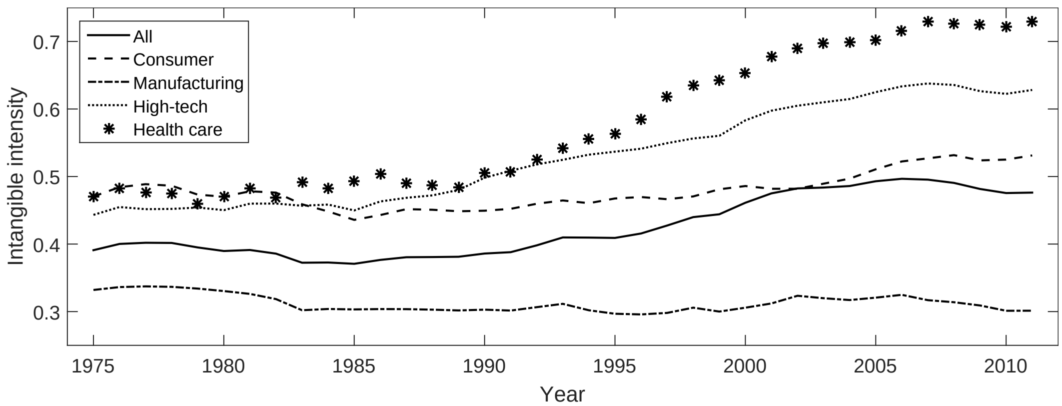
Statistics are based on the sample of Compustat firms from 1975 to 2011. The physical capital stock, K^{phy}, is measured as property, plant, and equipment (PP&E). We estimate the intangible capital stock, K^{int}, by applying the perpetual inventory method to firms' intangible investments, defined as research and development (R&D) and 0.3 $\times$ selling, general, and administrative (SG&A) spending. We then add in firms' balance sheet intangibles. Intangible intensity equals $K^{int}/(K^{int} + K^{phy})$. Knowledge capital is the part of intangible capital that comes from R&D. The denominator for all new measures is $K^{int} + K^{phy}$. The denominator for all standard measures is K^{phy}. The numerator for both q variables is the market value of equity plus the book value of debt minus current assets. The numerator for ι^{phy} is capital expenditure (CAPX), and the numerator for ι^{int} is R&D + (0.3 $\times$ SG&A). Total investment $\iota^{tot} = \iota^{phy} + \iota^{int}$. The numerator for standard cash flow is income before extraordinary items plus depreciation expenses. The numerator for total cash flow is the same but adds back intangible investment net a tax adjustment.

Variable	Mean	Median	Standard deviation	Skewness
Intangible capital stock (millions of dollars)	427	41.7	1990	11.6
Physical capital stock (millions of dollars)	1237	77.9	6691	16.5
Intangible intensity	0.43	0.45	0.27	-0.01
Knowledge capital / intangible capital	0.24	0.01	0.37	1.65
New measures				
Total q (q^{tot})	1.11	0.57	1.91	3.76
Physical investment (ι^{phy})	0.10	0.06	0.14	3.47
Intangible investment (ι^{int})	0.11	0.09	0.11	1.92
Total investment (ι^{tot})	0.21	0.16	0.18	2.61
Total cash flow (c^{tot})	0.16	0.15	0.19	0.52
Standard measures				
Standard q (q^*)	3.14	0.93	7.22	4.41
CAPX/PPE (ι^*)	0.19	0.11	0.24	3.52
Standard cash flow (c^*)	0.15	0.16	0.62	-1.63

R&D. The average q^{tot} is mechanically smaller than q^*, because its denominator is larger. The gap is dramatic in some cases. For example, Google's q^* is 10.1 in 2013, but its q^{tot} is only 3.2. Researchers sometimes discard q observations exceeding 10, arguing they are unrealistically large. Total q exceeds 10 in only 1% of observations, compared with 7% for standard q, suggesting total q is a more reliable measure. The standard deviation of q^{tot} is 74% lower than for q^*. The standard deviation scaled by its mean is also lower. The average physical and intangible investment rates are roughly equal, but physical investment is more volatile and right-skewed.

Fig. 1 shows that the average intangible intensity has increased over time, especially in the 1990s. The figure also shows that high-tech and health firms are heavy users of intangible

Figure 7: Capital intangibility over time



This figure plots the mean intangible capital intensity over time, both for our full sample and within industries. Intangible intensity equals $K^{int}/(K^{int}+K^{phy})$, the firm's stock of intangible divided by its total stock of capital. We use the Fama and French five-industry definition and exclude industry "Other."

capital and that manufacturing firms use less. Somewhat surprisingly, even manufacturing firms' capital is 30–34% intangible on average.

2.4. Full-sample results

In this section, we test the theory's predictions in our full sample. Section 5 compares results across subsamples. We begin with the classic OLS panel regressions of Fazarri, Hubbard, and Petersen (1988). We then correct for measurement error bias in Subsection 2.4.2.

2.4.1. OLS results and co-movement in investment

Table 12 contains results from OLS regressions of investment on lagged q and firm and year fixed effects. The columns compare different investment measures. For now, we focus on R^2 values, because the regression coefficients suffer from measurement error bias. This bias is especially severe for cash flow coefficients (Erickson and Whited, 2000; Abel, 2014), so we exclude cash flow until Subsection 4.2.

Taken literally, the theory predicts an R^2 of 100% in Panel A when we measure investment as ι^{phy}, ι^{int}, or ι^{tot}. We find R^2 values that are well below 100%. One potential explanation is that we measure q with error, an issue we address in Subsection 4.2. Another is that

Table 12: Ordinary least squares results

Results are from OLS panel regressions of investment on lagged Tobin's q and firm and year fixed effects. Each column uses a different investment measure. Physical investment (ι^{phy}) equals capital expenditure (CAPX) scaled by total capital ($K^{tot} = K^{phy} + K^{int}$). Intangible investment (ι^{int}) equals research and development (R&D) + 0.3 × selling, general and administrative (SG&A) expense, scaled by K^{tot}. Total investment equals $\iota^{phy} + \iota^{int}$. R&D investment equals R&D scaled by total capital. The R&D column excludes observations with missing R&D. The investment measure in the final column is CAPX divided by property, plant and equipment (PP&E). Panel A shows regressions on total q, denoted q^{tot}. Panel B shows regressions on standard q, denoted q^*. The numerator for both q variables is the market value of equity plus the book value of debt minus current assets. The denominator for q^{tot} is K^{tot}. The denominator for q^* is K^{phy}. Standard errors clustered by firm are in parentheses. We report the within-firm R^2. Panel C tests whether the R^2 values in Panels A and B are different, taking into account the correlation across regressions and again clustering by firm. Data are from Compustat from 1975 to 2011.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	CAPX/PPE (ι^*)
Panel A: Regressions with total q					
Total q	0.029	0.020	0.049	0.013	0.062
	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
R^2	0.209	0.279	0.327	0.270	0.244
	(0.008)	(0.007)	(0.006)	(0.009)	(0.008)
Panel B: Regressions with standard q					
Standard q	0.006	0.005	0.011	0.003	0.017
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.139	0.266	0.250	0.250	0.233
	(0.009)	(0.008)	(0.007)	(0.010)	(0.008)
Panel C: Difference in R^2 (Panel A − Panel B)					
ΔR^2	0.070	0.013	0.077	0.020	0.011
	(0.003)	(0.004)	(0.003)	(0.005)	(0.003)
Number of observations	141,800	141,800	141,800	75,426	141,800

slopes vary across firms or that shocks hit firms' marginal adjustment cost functions. Our theory's prediction holds better for intangible investment ($R^2 = 27.9\%$) than for physical investment ($R^2 = 20.9\%$), and it holds better still for total investment ($R^2 = 32.7\%$). We also check that this result holds for the portion of intangible investment coming from R&D, because the portion from SG&A is measured with more error. When we measure investment as R&D scaled by total capital, we find an R^2 of 27.0%, which is similar to the 27.9% R^2 from our main intangible investment measure, ι^{int}.

Our theory predicts a lower R^2 for the literature's usual regression of CAPX/PPE on standard q, shown in the last column of Panel B. The R^2 here is low (23.3%) relative to all the R^2 values in Panel A, with one exception: Standard q explains standard investment slightly better than total q explains our new physical investment measure, ι^{phy}. For ι^{phy}, measurement error in intangible capital could be offsetting any improvements from including intangible capital in the denominator of q.

One interesting implication of our theory is that physical and intangible investment should co-move strongly within firms, because the two capital types have the same marginal productivity and, hence, the same marginal q. We find strong co-movement in the data: ι^{phy} and ι^{int} have a 31% correlation after we remove firm and time fixed effects from both. According to the theory, this co-movement should decrease if we remove the effects of total q, for example, by isolating the residuals for ι^{phy} and ι^{int} from Panel A. The correlation between these two regressions' residuals is lower (17%), consistent with the theory. This remaining correlation could just be an artifact of measurement error in total q.

Throughout the corporate finance literature, researchers use Tobin's q to proxy for firms' investment opportunities. The R^2 values in Table 2 help us judge how well these proxies work and, in particular, whether total q or the literature's standard q measure is the better proxy for investment opportunities. Panel B shows how well standard q explains the five investment measures, and Panel C tests whether total q or standard q delivers a higher R^2.[7]

[7]Throughout, we conduct inference on R^2 values using influence functions (Newey and McFadden, 1994).

For all five investment measures, total q delivers a larger R^2 value than standard q. The improvement in R^2 ranges from 1 to 8 percentage points, or from 5% to 50%. Some of the improvements are modest in magnitude, but statistical significance in Panel C is high, with t-statistics ranging from 3.4 to 25.

It is tempting to run a horse race by including total and standard q in the same regression. Because both variables proxy for q with error, their resulting slopes would be biased in an unknown direction, making the results difficult to interpret (Klepper and Leamer, 1984). For this reason, we do not tabulate results from such a horse race. We simply note that regressing either ι^{phy} or ι^{tot} on both q proxies produces a positive and highly significant slope on q^{tot} but a negative and less significant slope on q^*. For ι^{int} and ι^*, both q variables have a significantly positive slope, but the slope on q^{tot} is much larger in magnitude.

Outside the investment literature, it is popular to measure Tobin's q as the firm's market value scaled by its book value of assets. Like Erickson and Whited (2006, 2012), we find that these market-to-book–assets ratios are especially poor proxies for investment opportunities. They produce lower R^2 values than both standard and total q no matter how we measure investment (Online Appendix, Table A1).

To summarize, total q explains intangible investment slightly better than physical investment in our full sample, and it explains total investment even better. As our theory predicts, physical and intangible investment co-move strongly within firms, because they share the same q. This result suggests strong co-movement between physical and intangible capital's marginal productivities. Judging by these results, the neoclassical theory of investment is just as relevant for intangible capital as it is for physical capital. We also show that total q is a superior proxy for investment opportunities no matter how investment is measured.

In a regression $y = \beta x + \epsilon$, this approach takes into account the estimation error in β, $var(y)$, and $var(x)$. We cluster by firm, which accounts for autocorrelation both within and across regressions.

2.4.2. Bias-corrected results

According to our theory, total q is better than standard q at approximating the true, unobservable q. We recognize, however, that total q is still a noisy proxy. For one, we measure intangible capital with error. Also, Tobin's q measures average q, but investment depends on marginal q in theory. Average q equals marginal q in our simple theory, but, to the extent that reality departs from this theory, average q measures marginal q with error.[8]

Because we have only a proxy for q, all the OLS slopes from Subsection 4.1 suffer from measurement error bias. We now estimate the previous models while correcting this bias using the Erickson, Jiang, and Whited (2014) higher-order cumulant estimator.[9] The cumulant estimator provides unbiased estimates of β in the following classical errors-in-variables model:

$$\iota_{it} = a_i + q_{it}\beta + z_{it}\alpha + u_{it} \tag{2.16}$$

$$p_{it} = \gamma + q_{it} + \varepsilon_{it}, \tag{2.17}$$

where p is a noisy proxy for the true, unobservable q and z is a vector of perfectly measured control variables. The cumulant estimator's main identifying assumptions are that p has nonzero skewness, $\beta \neq 0$, and u and ε are independent of q, z, and each other.

Because the cumulant estimator corrects for measurement error, why is a new q proxy with less measurement error needed? The reason is that, by ignoring intangibles, the literature's standard physical investment and q proxies, ι^* and q^*, are both mismeasured, and the measurement error is multiplicative, not additive. The measurement error in ι^* and q^* comes

[8] Gala (2014) measures the differences between marginal and average q.

[9] The cumulant estimator supersedes the Erickson and Whited (2002) higher-order moment estimator. Cumulants are polynomials of moments. The estimator is a generalized method of moments (GMM) estimator with moments equal to higher-order cumulants of investment and q. Compared with the Erickson and Whited (2002) estimator, the cumulant estimator has better finite-sample properties and a closed-form solution, which makes numerical implementation easier and more reliable. We use the third-order cumulant estimator, which dominates the fourth-order estimator in the estimation of τ^2 (Erickson and Whited, 2012; Erickson, Jiang, and Whited, 2014). Results are similar using the fourth-order cumulant estimator (Online Appendix).

from the omission of intangibles. Because that same error is in both variables, their measurement errors are correlated with each other, violating the cumulant estimator's assumption that u and ε are independent.[10] The measurement error is multiplicative, because changing the variables' denominators from physical to total capital requires multiplying them both by K^{phy}/K^{tot}. The multiplicative error makes both variables' errors depend on the true q, violating the cumulant estimator's assumption that u and ε are independent of q. We cannot solve the problem by regressing total investment on the standard q measure (ι^{tot} on q^*), because measurement error in q^* is still multiplicative and, hence, a function of q, again violating the cumulant estimator's assumption that ε is independent of q.[11]

We perform a simple horse race to illustrate that the cumulant estimator on its own cannot correct for the measurement error in the standard q measure. Using the cumulant estimator, we regress ι^{tot} on q^*, and then regress ι^{tot} on q^{tot}. If the cumulant estimator could correct for the measurement error in q^*, then the two q proxies should produce similar q-slope estimates and ρ^2 values (defined below). Instead, we find that using total q produces a significantly higher q-slope (0.086 versus 0.023) and higher ρ^2 (0.423 versus 0.314). Results are in the Online Appendix.

Estimation results are in Table 13. Regarding the slopes on q, our estimates imply that intangible capital's convex adjustment costs are roughly twice as large as those for physical capital. According to our theory, the q-slopes measure the inverse capital adjustment-cost parameters γ^{phy} and γ^{int}. In Panel A, the 0.070 slope for ι^{phy} is roughly double the 0.037 slope for ι^{int}. We obtain a similar result after controlling for cash flow (Panel B) and also if we isolate the R&D component of intangible investment (Column 4). As we explain in Subsection 2.2, an important caveat is that our regressions can identify the

[10]To see this, assume that the world behaves according to $\iota_{it}^{tot} = q_{it}\widetilde{\beta}$, where q_{it} is the unobservable, true q; that our empirical proxy $q_{it}^{tot} = q_{it} + \widetilde{\varepsilon}_{it}$, where $\widetilde{\varepsilon}_{it}$ is independently distributed; and that we mistakenly estimate the errors-in-variables model using the standard measures: $\iota_{it}^{*} = q_{it}\beta + u_{it}$ and $q_{it}^{*} = q_{it} + \varepsilon_{it}$. One can prove that $u_{it} = q_{it}\left(A_{it}B_{it}\widetilde{\beta} - \beta\right)$ and $\varepsilon_{it} = q_{it}\left(B_{it} - 1\right) + B_{it}\widetilde{\varepsilon}_{it}$, where $B_{it} = K_{it}^{tot}/K_{it}^{phy}$ and $A_{it} = I_{it}^{phy}/I_{it}^{tot}$. Because u_{it} and ε_{it} both depend on $q_{it}B_{it}$, they are not independent of q or each other.

[11]To see this, suppose that the assumptions in footnote 10 hold, except we instead estimate the errors-in-variables model $\iota_{it}^{tot} = q_{it}\beta + u_{it}$, and $q_{it}^{*} = q_{it} + \varepsilon_{it}$. One can prove that $\varepsilon_{it} = q_{it}\left(B_{it} - 1\right) + \widetilde{\varepsilon}_{it}B_{it}$, so ε_{it} and q_{it} are not independent of each other.

convex component but not the overall level of adjustment costs.

This result helps support the Brown, Fazzari, and Petersen (2009, p. 160) conjecture that "R&D likely has high adjustment costs ..., possibly substantially higher than the adjustment costs for physical investment." Their argument is that R&D involves spending on highly skilled technology workers who are costly to hire, train, and replace. Hall (2002), Himmelberg and Petersen (1994), Griliches and Hausman (1986), and Grabowski (1968) make similar arguments about R&D, and one could make similar arguments about human capital investments that are part of SG&A. Empirical evidence supporting these arguments is currently very limited.[12] An important implication of our result is that firms adjust more slowly to shocks to their investment opportunities as the economy shifts toward intangible capital.

Table 13 also changes how we view physical capital's adjustment costs. The last column in Panel A shows the literature's standard regression, which omits intangible capital. Prediction 4 in our theory states that this regression delivers a downward-biased estimate of $1/\gamma^{phy}$, i.e., a downward-biased q-slope. Consistent with this prediction, this standard regression delivers a q-slope of 0.036, roughly half as large as the 0.070 slope from the regression using ι^{phy} and scaling q by total capital. As we explain in Subsection 2.1, the typical regression delivers downward-biased slopes because the ratio of physical to total capital is an omitted variable that it positively related to the regressor and negatively related to the residual. This result helps resolve a puzzle in the investment literature. Researchers since Summers (1981) have argued that investment-q regressions produce implausibly small q-slopes, i.e., large adjustment costs. We find that physical capital's q-slopes are twice as large as previously believed, once one accounts for intangible capital.

[12] Bernstein and Nadiri (1989a, 1989b) and Mohnen, Nadiri, and Prucha (1986) report slower adjustment speeds for R&D capital than physical capital in most but not all industries and for most but not all of their adjustment-cost measures. Bernstein and Nadiri (1989a, 1989b) use data on 48 firms from 1965 to 1978 and 35 firms from 1959 to 1966, respectively. Mohnen, Nadiri, and Prucha (1986) use three countries' aggregate data from 1965 to 1977. Intangible capital was less prevalent during these years, and US firms were not required to report R&D until 1975. Li, Liu, and Xue (2014) estimate a structural model off the cross section of stock returns, and they find larger adjustment costs for R&D capital. By focusing on R&D capital, all these papers exclude organization capital.

Table 13: Bias-corrected results

Results are from regressions of investment on lagged Tobin's q, firm fixed effects, and (in Panel B) contemporaneous cash flow, all estimated using the cumulant estimator. Each column uses a different investment measure as defined in Table 12. Total (standard) q equals the firm's market value scaled by K^{tot} (K^{phy}). The numerator for standard cash flow is income before extraordinary items plus depreciation expenses. The numerator for total cash flow is the same but adds back intangible investment net a tax adjustment. Total cash flow is scaled by K^{tot}; standard cash flow, by K^{phy}. ρ^2 is the within-firm R^2 from a hypothetical regression of investment on true q, and τ^2 is the within-firm R^2 from a hypothetical regression of our q proxy on true q. For comparison, the table also shows the ordinary least squares (OLS) R^2 values from Table 12. Standard errors clustered by firm are in parentheses. Data are from Compustat from 1975 to 2011.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	CAPX/PPE (ι^*)
Panel A: Regressions without cash flow					
Total q (q^{tot})	0.070	0.037	0.086	0.023	
	(0.001)	(0.001)	(0.001)	(0.000)	
Standard q (q^*)					0.036
					(0.001)
OLS R^2	0.209	0.279	0.327	0.270	0.233
	(0.008)	(0.007)	(0.006)	(0.009)	(0.008)
ρ^2	0.358	0.392	0.423	0.376	0.372
	(0.008)	(0.008)	(0.008)	(0.011)	(0.008)
τ^2	0.437	0.559	0.597	0.593	0.492
	(0.009)	(0.012)	(0.010)	(0.016)	(0.010)
Panel B: Regressions with cash flow					
Total q (q^{tot})	0.069	0.038	0.086	0.024	
	(0.001)	(0.001)	(0.002)	(0.001)	
Total cash flow (c^{tot})	0.024	0.050	0.140	0.000	
	(0.008)	(0.004)	(0.009)	(0.004)	
Standard q (q^*)					0.035
					(0.001)
Standard cash flow (c^*)					0.015
					(0.004)
OLS R^2	0.235	0.326	0.374	0.281	0.233
	(0.008)	(0.007)	(0.006)	(0.009)	(0.008)
ρ^2	0.361	0.447	0.481	0.405	0.371
	(0.008)	(0.009)	(0.008)	(0.011)	(0.008)
τ^2	0.435	0.502	0.544	0.568	0.494
	(0.010)	(0.014)	(0.011)	(0.017)	(0.011)
Number of observations	141,800	141,800	141,800	75,426	141,800

In addition to delivering unbiased q-slopes, the cumulant estimator produces two useful test statistics. The first, ρ^2, is the hypothetical R^2 from Eq. (2.16). Loosely speaking, ρ^2 indicates how well the true, unobservable q explains investment, with $\rho^2 = 1$ implying a perfect relation. Taken literally, our theory predicts $\rho^2 = 1$ even if we measure q with error. The second statistic, τ^2, is the hypothetical R^2 from Eq. (2.17). It indicates how well our q proxy explains true q, with $\tau^2 = 1$ implying a perfect proxy.

Comparing the total-investment regression with the literature's typical regression, we find that including intangible capital produces a stronger investment-q relation (ρ^2 of 0.423 versus 0.372, a 14% increase) and a better proxy for Tobin's q (τ^2 of 0.597 versus 0.492, a 21% increase). On these dimensions, the classic q-theory fits the data better when we account for intangible capital. Model fit is still far from perfect, though: q explains less than half the variation in investment, and our total-q proxy explains less than 60% of the variation in true q.

Finally, we discuss the cash flow slopes shown in Panel B. Our simple theory predicts a zero cash flow slope for regressions using ι^{phy}, ι^{int}, and ι^{tot}. We find that physical investment has a significantly positive cash flow slope, contrary to the theory's prediction. We also find that including intangible capital affects the sensitivity of physical investment to cash flow. The physical investment–cash flow sensitivity is 60% higher (0.024 versus 0.015) when we compare the specification with ι^{phy} with the standard regression using CAPX/PPE.

Compared with physical investment, intangible investment appears roughly twice as sensitive to cash flow (slope of 0.050 versus 0.024). Intangible investment is the sum of its R&D and SG&A components. Which component is most important for producing the high investment–cash flow sensitivity? Column 4 of Panel B shows that R&D investment has a slope of zero on cash flow, consistent with the theory's prediction. SG&A investment must be highly sensitive to cash flow. Indeed, we find that SG&A investment has a cash flow slope of 0.115, which is more than double intangible investment's slope of 0.050 (Online Appendix).

One concern here is that measurement error in SG&A investment is biasing its cash flow slope upward. Compared with R&D, we measure SG&A investment with considerable error. Our c^{tot} measure is gross of SG&A investment, meaning we add back SG&A investment when computing it [Eq. (2.15)]. Any measurement error in SG&A investment, therefore, appears mechanically in both c^{tot} and SG&A investment itself, biasing its cash flow slope upward. For this reason, we view 0.115 as an upper bound for SG&A investment's cash flow sensitivity. We provide a lower bound in the Online Appendix by creating an alternate cash flow measure that is net of SG&A and, therefore, immune from this concern. SG&A investment has a statistically insignificant slope of 0.008 on this alternate cash flow measure. This 0.008 slope provides a lower bound for the true slope, because netting SG&A from cash flow pushes down the cash flow slope, and an economically meaningful cash flow measure should be gross of all investment, including SG&A investment. In sum, we can provide only a wide range for SG&A investment's cash flow slope (0.008–0.115), which implies a wide range in intangible investment's cash flow slope (0.012–0.050). Whether physical or intangible investment is more sensitive to cash flow is unclear.

Even absent these measurement challenges, interpreting the investment–cash flow sensitivity is notoriously difficult. Fazzari, Hubbard, and Petersen (1988) interpret it as evidence of financing constraints. In contrast, theories by Gomes (2001), Alti (2003), Cooper and Ejarque (2003), Hennessy and Whited (2007), Abel and Eberly (2011), and Gourio and Rudanko (2014) predict an investment–cash flow sensitivity even in the absence of financing constraints. For example, decreasing returns to scale can make cash flow informative about marginal q, even after controlling for Tobin's (average) q. We simply conclude that physical investment is even more sensitive to cash flow than previously believed, R&D investment is insensitive to cash flow, and SG&A investment's cash flow sensitivity remains unclear. Without performing a full structural estimation, it is difficult to tell whether these cash flow results are driven by financing constraints, diseconomies of scale, or some other source.

2.5. Comparing subsamples

Next, we compare results across firms, industries, and years. Doing so allows us to test our theory and compare adjustment costs across subsamples. It also lets us check our main results' robustness across subsamples, which we discuss in Section 2.7.

We reestimate the previous models in subsamples formed using three variables. First, we sort firms each year into quartiles based on their lagged intangible intensity (Table 14). Second, we use the Fama and French five-industry definition to compare the manufacturing, consumer, high-tech, and health industries (Table 15). Third, we compare the early (1972–1995) and late (1996–2011) parts of our sample (Table 16). For each subsample, we estimate regressions using ι^{phy}, ι^{int}, and ι^{tot}, as well as the standard regression with CAPX/PPE.

2.5.1. Testing the theory in subsamples

The classic q theory, including the theory in this paper, fits the data better in settings with more intangible capital. We find this improved fit on three dimensions.

First, R^2 values increase dramatically when moving from the lowest to highest intangible quartile (Table 14, Panel B). For example, the R^2 for the total-investment regression increases monotonically from 23% to 47%. Even when we use the literature's standard investment and q measures, the R^2 increase monotonically from 18% to 30%. This last result is surprising, because the standard q measure has more measurement error in firms with more intangibles: τ^2 is 44% in Quartile 4 versus 68% in Quartile 1. The patterns are similar when we compare manufacturing with high-intangible industries or the early with the late subperiod. The increases in R^2 across subsamples, tabulated in the last columns of Tables 14–16, are statistically significant for all four investment measures and in all three tables, with just two exceptions out of 12 (Table 15, Specification 1 and Table 16, Specification 1).

Second, ρ^2 values increase monotonically and roughly double when moving from the lowest to highest intangible quartile (Panel C). This result means that the true q's explanatory

Table 14: Comparing firms with different amounts of intangible capital

This table shows results from subsamples formed based on yearly quartiles of intangible intensity, which equals the ratio of a firm's intangible to total capital. The first row show each quartile's mean intangible intensity. Results are from regressions of investment on lagged q, firm fixed effects, and (in Panel E) contemporaneous cash flow. Slopes on q and cash flow, as well as ρ^2 and τ^2 values, are from the cumulant estimator. R^2 is from the ordinary least squares (OLS) estimator that includes year fixed effects. Specifications 1–3 use physical (ι^{phy}), intangible (ι^{int}), and total investment (ι^{tot}), respectively, along with total q, all of which are scaled by total capital. Specification 4 uses standard investment ($\iota^* =$ capital expenditure (CAPX) / property plant and equipment (PPE)) and standard q (q^*), which is scaled by physical capital. The last row in Panel A shows the ratio of the Specification 2 q-slope to the sum of slopes from Specifications 1 and 2. We conduct inference using the delta method. Specifications 5–8 in Panel E add standard cash flow (c^*) and total cash flow (c^{tot}), defined in Table 13. Standard errors clustered by firm are in parentheses. We use influence functions to conduct inference for ρ^2 and τ^2.

Specification	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Quartile 4 − 1: Difference	Quartile 4 − 1: Standard error
Intangible intensity	8%	33%	56%	76%		
Panel A: Slopes on q						
(1) ι^{phy} on q^{tot}	0.095	0.081	0.063	0.050	-0.045	(0.004)
(2) ι^{int} on q^{tot}	0.027	0.032	0.035	0.038	0.011	(0.004)
(3) ι^{tot} on q^{tot}	0.101	0.097	0.086	0.074	-0.027	(0.004)
(4) CAPX/PPE on q^*	0.065	0.052	0.035	0.033	-0.032	(0.007)
$\beta^{int}/(\beta^{int}+\beta^{phy})$	22%	28%	36%	43%	21%	(2.90%)
Panel B: OLS R^2 values						
(1) ι^{phy} on q^{tot}	0.219	0.227	0.259	0.284	0.065	(0.026)
(2) ι^{int} on q^{tot}	0.061	0.170	0.306	0.458	0.397	(0.064)
(3) ι^{tot} on q^{tot}	0.232	0.270	0.357	0.473	0.241	(0.016)
(4) CAPX/PPE on q^*	0.182	0.195	0.248	0.299	0.117	(0.022)
Panel C: ρ^2 values						
(1) ι^{phy} on q^{tot}	0.261	0.364	0.486	0.612	0.351	(0.027)
(2) ι^{int} on q^{tot}	0.271	0.311	0.377	0.411	0.140	(0.048)
(3) ι^{tot} on q^{tot}	0.274	0.388	0.498	0.543	0.269	(0.020)
(4) CAPX/PPE on q^*	0.197	0.282	0.379	0.561	0.364	(0.023)
Panel D: τ^2 values						
(1) ι^{phy} on q^{tot}	0.650	0.478	0.374	0.375	-0.275	(0.033)
(2) ι^{int} on q^{tot}	0.196	0.365	0.561	0.792	0.596	(0.031)
(3) ι^{tot} on q^{tot}	0.664	0.519	0.503	0.659	-0.005	(0.034)
(4) CAPX/PPE on q^*	0.682	0.514	0.483	0.439	-0.243	(0.062)
Panel E: Slopes on cash flow						
(5) ι^{phy} on q^{tot}, c^{tot}	0.203	0.090	-0.009	-0.036	-0.239	(0.023)
(6) ι^{int} on q^{tot}, c^{tot}	-0.018	0.018	0.060	0.110	0.128	(0.013)
(7) ι^{tot} on q^{tot}, c^{tot}	0.227	0.148	0.100	0.129	-0.098	(0.024)
(8) CAPX/PPE on q^*, c^*	0.182	0.072	0.011	-0.003	-0.185	(0.026)
Number of observations	35,438	35,453	35,442	35,467		

Table 15: Comparing industries

This table shows results from industry subsamples. We use the Fama and French five-industry definition, excluding the industry "Other." Remaining details are the same as in Table 14.

					Quartile $4-1$	
Specification	Manufacturing	Consumer	High-tech	Health	Difference	Standard error
Intangible intensity	31%	48%	55%	62%		
Panel A: Slopes on q						
(1) ι^{phy} on q^{tot}	0.083	0.085	0.059	0.068	-0.015	(0.005)
(2) ι^{int} on q^{tot}	0.038	0.037	0.036	0.040	0.002	(0.003)
(3) ι^{tot} on q^{tot}	0.097	0.102	0.079	0.084	-0.013	(0.005)
(4) CAPX/PPE on q^*	0.041	0.042	0.033	0.038	-0.003	(0.003)
Panel B: OLS R^2 values						
(1) ι^{phy} on q^{tot}	0.194	0.239	0.307	0.244	0.050	(0.038)
(2) ι^{int} on q^{tot}	0.206	0.209	0.407	0.281	0.075	(0.031)
(3) ι^{tot} on q^{tot}	0.258	0.310	0.460	0.362	0.104	(0.024)
(4) CAPX/PPE on q^*	0.186	0.214	0.354	0.258	0.072	(0.031)
Panel C: ρ^2 values						
(1) ι^{phy} on q^{tot}	0.254	0.397	0.540	0.551	0.297	(0.036)
(2) ι^{int} on q^{tot}	0.321	0.234	0.474	0.376	0.055	(0.030)
(3) ι^{tot} on q^{tot}	0.294	0.386	0.572	0.521	0.227	(0.028)
(4) CAPX/PPE on q^*	0.206	0.290	0.549	0.545	0.339	(0.029)
Panel D: τ^2 values						
(1) ι^{phy} on q^{tot}	0.557	0.442	0.431	0.319	-0.238	(0.035)
(2) ι^{int} on q^{tot}	0.398	0.485	0.686	0.545	0.147	(0.041)
(3) ι^{tot} on q^{tot}	0.632	0.539	0.634	0.522	-0.110	(0.036)
(4) CAPX/PPE on q^*	0.655	0.539	0.511	0.365	-0.290	(0.048)
Panel E: Slopes on cash flow						
(5) ι^{phy} on q^{tot}, c^{tot}	0.171	0.029	-0.033	-0.059	-0.230	(0.032)
(6) ι^{int} on q^{tot}, c^{tot}	0.041	0.106	0.059	-0.019	-0.060	(0.017)
(7) ι^{tot} on q^{tot}, c^{tot}	0.265	0.190	0.090	0.010	-0.255	(0.034)
(8) CAPX/PPE on q^*, c^*	0.083	0.048	0.001	-0.003	-0.086	(0.018)
Number of observations	40,280	36,884	31,680	11,207		

Table 16: Comparing time periods

This table shows results from the early (1975–1995) and late (1996–2011) subsamples. The 1995 breakpoint produces subsamples of roughly equal size. Remaining details are the same as in Table 14.

			Late − Early	
Specification	Early	Late	Difference	Standard error
Intangible intensity	39%	47%		
Panel A: Slopes on q				
(1) ι^{phy} on q^{tot}	0.083	0.062	-0.021	(0.002)
(2) ι^{int} on q^{tot}	0.035	0.037	0.002	(0.002)
(3) ι^{tot} on q^{tot}	0.100	0.079	-0.021	(0.004)
(4) CAPX/PPE on q^*	0.043	0.033	-0.010	(0.001)
Panel B: OLS R^2 values				
(1) ι^{phy} on q^{tot}	0.205	0.208	0.003	(0.018)
(2) ι^{int} on q^{tot}	0.190	0.328	0.138	(0.016)
(3) ι^{tot} on q^{tot}	0.273	0.357	0.084	(0.013)
(4) CAPX/PPE on q^*	0.209	0.268	0.059	(0.017)
Panel C: ρ^2 values				
(1) ι^{phy} on q^{tot}	0.304	0.407	0.103	(0.016)
(2) ι^{int} on q^{tot}	0.259	0.497	0.238	(0.018)
(3) ι^{tot} on q^{tot}	0.336	0.511	0.175	(0.016)
(4) CAPX/PPE on q^*	0.262	0.479	0.217	(0.016)
Panel D: τ^2 values				
(1) ι^{phy} on q^{tot}	0.501	0.423	-0.078	(0.022)
(2) ι^{int} on q^{tot}	0.504	0.584	0.080	(0.030)
(3) ι^{tot} on q^{tot}	0.595	0.603	0.008	(0.022)
(4) CAPX/PPE on q^*	0.615	0.477	-0.138	(0.026)
Panel E: Slopes on cash flow				
(5) ι^{phy} on q^{tot}, c^{tot}	0.109	-0.033	-0.142	(0.017)
(6) ι^{int} on q^{tot}, c^{tot}	0.090	-0.033	-0.123	(0.013)
(7) ι^{tot} on q^{tot}, c^{tot}	0.256	0.038	-0.218	(0.020)
(8) CAPX/PPE on q^*, c^*	0.074	-0.008	-0.082	(0.009)
Number of observations	69,753	72,047		

power for investment is much stronger in firms with more intangible capital. This increase in ρ^2 is responsible for the large increases in R^2 across subsamples. Again, the patterns are similar across industries and years.

Third, cash flow slopes are significantly lower in firms, industries, and years with more intangible capital (Panel E). The cash flow slopes even turn slightly negative in several high-intangible subsamples, even when we use the literature's standard measures. This result is robust across all four investment measures and across Tables 14–16, with one exception: Intangible investment has a larger cash flow slope in higher-intangible quartiles (Table 14). This exception could be an artifact of the measurement error bias we discuss in Subsection 4.2. Like us, Chen and Chen (2012) find a weaker investment-cash flow sensitivity in recent years. Our findings suggest this change over time could partially reflect the rise of intangible capital.

The rest of this subsection seeks to explain why the classic q theory works better in settings with more intangibles. Put differently, which of the theory's assumptions are violated more severely in firms using less intangibles? We start by exploring theoretically whether violations of our simple model's assumptions could explain the patterns in Table 14. We solve a more general model that relaxes our earlier assumptions about constant returns to scale, perfect competition, and quadratic adjustment costs. Details and numerical results are in the Online Appendix. We explain two predictions from the model next.

We find that violating the assumption about quadratic adjustment costs is unlikely to generate the empirical patterns in Table 14. When we change the adjustment-cost function's exponent from 2 to 1.75 or 1.5, we find a negligible effect on predicted R^2 values, and we do not find a significant predicted investment–cash flow relation.

Differences in economies of scale or competition could theoretically explain some of the patterns in Table 14. Relative to the benchmark theory in Section 2, a theory with imperfect competition or decreasing returns to scale produces lower predicted R^2 values in regressions of investment on q, and it also generates a positive investment–cash flow relation, a prediction already known from Abel and Eberly (2011). If firms using more intangible capital are closer to the perfect-competition, constant-returns benchmark, this mechanism could explain why they exhibit lower cash flow slopes and higher R^2 and ρ^2 values.

Unfortunately, we find little empirical support for this mechanism. In the Online Appendix, we check whether firms with more intangibles are closer to the perfect-competition, constant-returns benchmark. First, we estimate production-function curvature using the methods of Cooper and Haltiwanger (2006) and Olley and Pakes (1996). Comparing the curvature estimates across subsamples, we find no statistically significant differences in economies of scale between the high- and low-intangible quartiles. Also, whereas the improvement in model fit in Table 14 is monotonic across quartiles, the curvature estimates are strongly non-monotonic. Second, we compare three competition proxies across subsamples. We find mixed results when we use the Herfindahl Index to proxy for industry-level competition; different industry classifications deliver increasing, decreasing, or flat patterns across intangible-intensity subsamples. We also compare profitability across subsamples, because competition should reduce profitability. Again, different profitability measures produce opposing results. We also compare firm size across subsamples, as relatively small firms within an industry can face more competition. The relation between firm size and intangible usage is either statistically insignificant or strongly non-monotonic depending on the size proxy we use. To summarize, we do not find any robust empirical evidence that high-intangible firms face less diseconomies of scale or more competition.

One last possible explanation for the pattern in Table 14 is that high-intangible firms are less financially constrained, making the theory fit the data better. This explanation seems unlikely, because it is difficult to use intangible assets as collateral, which arguably makes high-intangible firms more financially constrained (Almeida and Campello, 2007; Falato, Kadyrzhanova, and Sim, 2013). Unfortunately, it is difficult to test this financing-constraints mechanism without a full structural estimation (Hennessy and Whited, 2007).

2.5.2. Comparing adjustment costs across subsamples

Table 14 shows interesting patterns in q-slopes across subsamples. According to our theory, these q-slopes do not help us test our theory's predictions or assumptions. Instead, the q-slopes reflect adjustment-cost parameters.

Table 14 shows that firms using more intangibles have significantly smaller slopes of physical investment on q, and they have significantly larger slopes of intangible investment on q. The implications are that firms using more intangibles have physical capital that exhibits larger convex adjustment costs and that they have intangible capital that exhibits smaller convex adjustment costs.

This pattern in q-slopes points to differences in the nature of physical and intangible capital across firms, and it could also shed light on why some firms use more intangible capital. As we explain at the end of Subsection 2.1, if a firm's intangible capital is less costly than physical capital to adjust, then the firm is predicted to use relatively more intangible capital. As a result, firms using more intangible capital should have a higher intangible investment q-slope relative to the sum of slopes for physical and intangible investment. We show these slope ratios in Panel A of Table 14. The ratios increase monotonically across the quartiles, consistent with our theory. Our theory further predicts that the slope ratio equals firms' intangible intensity. The actual intensities, shown in the column labels, range from 8% to 76%, and the slope ratios ranges from only 22% to 43%. Our simple theory, therefore, explains part but not all of firms' different intangible-capital usage.

This exercise provides a useful consistency check on our theory. Some important caveats are in order, though. To link q-slopes to firms' optimal mix of capital types, our theory needs strong additional assumptions. The theory requires that physical and intangible capital are identical in all ways except for their quadratic adjustment-cost parameters. Outside our simple theory, alternate explanations could exist for the pattern we find in q-slopes across firms. We know from the investment-q literature that q-slopes need not reflect adjustment costs. For example, Abel and Eberly (2011) show that, even in a world with no adjustment costs, diseconomies of scale can make investment and Tobin's q positively related. Also, differences could exist between physical and intangible capital's purchase prices, depreciation rates, economies of scale, and adjustment-cost curvatures. These differences could affect both firms' optimal mix of capitals and their investment-q slopes. Because Table 14 does not control for these differences, we might just be picking up these omitted differences between physical and intangible capital.

To explore this potential bias further, we solve a more general model that allows physical and intangible capital to differ in ways not allowed in Section 2. We assume that physical and intangible capital share the same adjustment-cost parameters ($\gamma^{phy} = \gamma^{int}$), so we shut down the mechanism proposed above. We then ask whether other differences between physical and intangible capital could produce predicted q-slope patterns like the ones in Table 14. Details and numerical results are in the Online Appendix. First, we find that differences in purchase prices ($p_{it}^{phy} \neq p_{it}^{int}$) can explain why some firms use more intangible capital, but they do not explain why firms have different investment-q slopes. Second, we show that differences between the two capital types' economies of scale do not necessarily drive them to use more of one capital type and do not make their q-slopes differ significantly. These first two alternate explanations—differences in purchase prices or economies of scale—do not seem to work for the empirical patterns we find. Third, we show that if intangible capital depreciates faster than physical capital, then firms optimally use less intangibles and intangible investment has a slightly lower q-slope than physical investment, consistent with the patterns in Table 14. Finally, we relax the assumption that both capital types face quadratic adjustment costs. We show that if intangible capital faces less-convex adjustment costs than physical capital, then firms optimally use less intangible capital and intangible investment has a lower q-slope than physical investment, consistent again with the patterns in Table 14. We cannot rule out that these last two mechanisms, differences in depreciation rates or adjustment-cost convexities between the two capital types, are driving the Table 14 cross-sectional relation between q-slopes and capital mixes.

2.6. Macro results

The neoclassical theory of investment, including the theory in this paper, can easily be interpreted as a theory of the macroeconomy, not a single firm. The macro literature has been interested in the investment-q relation going back to at least Abel (1980) and Summers (1981). We ask how this relation changes when we account for intangible capital.

Our macro sample contains 141 quarterly observations for the US economy from 1972Q2 to 2007Q2, the longest period for which all variables are available. Data on aggregate physical q and investment come from Hall (2001), who uses the flow of funds and aggregate stock and bond market data. The literature's standard q measure, again denoted q^*, is the ratio of the value of ownership claims on the firm, less the book value of inventories, to the reproduction cost of plant and equipment. The standard investment measure, again denoted ι^*, equals private nonresidential fixed investment scaled by its corresponding stock, both of which are from the Bureau of Economic Analysis.

Data on the aggregate stock and flow of physical and intangible capital come from Carol Corrado and are discussed in Corrado and Hulten (2014). Earlier versions of these data are used by Corrado, Hulten, and Sichel (2009) and Corrado and Hulten (2010). Their measures of intangible capital include aggregate spending on business investment in computerized information (from NIPA), R&D (from the National Science Foundation and Census Bureau), and economic competencies, which include investments in brand names, employer-provided worker training, and other items. One advantage of these macro data relative to our firm-level data is that the macro data do not rely on an assumption about the fraction of SG&A representing an investment. As before, we measure the total capital stock as the sum of the physical and intangible capital stocks. We compute total q as the ratio of total ownership claims on firm value, less the book value of inventories, to the total capital stock. We define the investment rates ι^{phy}, ι^{int}, and ι^{tot} as in our firm-level analysis. To mitigate problems from potentially differing data coverage, we use the Corrado and Hulten (2014) ratio of physical to total capital to adjust the Hall (2001) measures of physical q and investment.[13]

The correlation between standard and total q is extremely high, 0.997. The reason is that total q equals standard q times the ratio of physical to total capital, and this ratio has changed slowly and consistently over time (Fig. 1). Of more importance is the change from standard to total investment, which additionally requires multiplying ι^* by the ratio of capital flows, which is much more volatile than the ratio of capital stocks. The correlation between total and standard investment is therefore much smaller, 0.43.

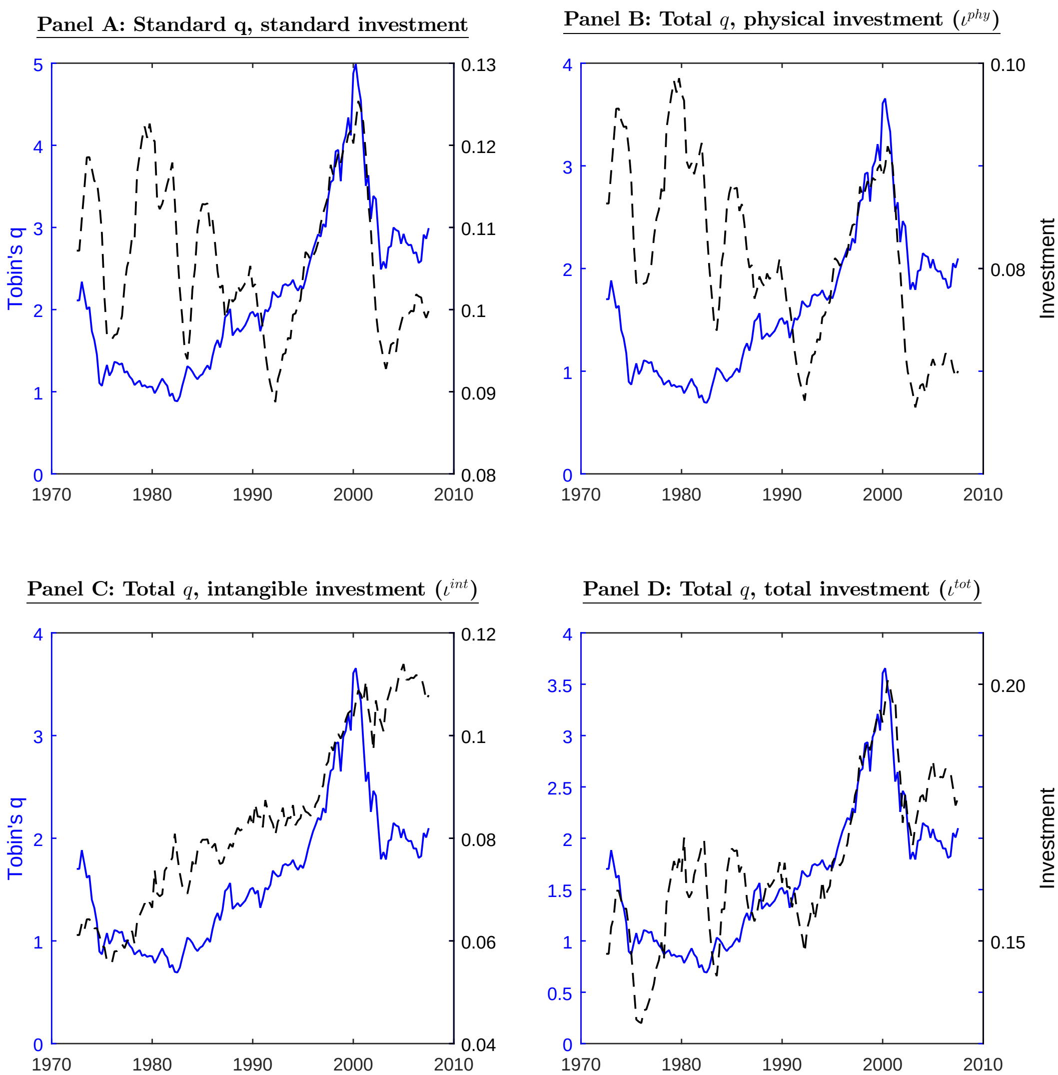
For comparison, we also use the Philippon (2009) aggregate bond q measure, which he obtains by applying a structural model to data on bond maturities and yields. Bond q is available at the macro level but not at the firm level. Philippon (2009) shows that bond q explains more of the aggregate variation in what we call physical investment than standard q does. Bond q data are from Philippon's website.

Fig. 2 plots the time series of investment and q. Panel A shows the standard q and investment measures, which omit intangible capital. Except in a few subperiods, q explains investment relatively poorly, as Philippon (2009) and others have shown. Panel B shows that the relation between total q and ι^{phy} is still weak. Panel C shows a strong relation between total q and intangible investment, mainly because total q and intangible investment both trend up from 1982 to 2000. Panel D compares total investment and total q. Here the fit looks strongest of all.

To explore these patterns more carefully, Table 17 shows results from time series regressions of investment on lagged q. Panel A shows regressions in levels, comparing our four investment measures. Consistent with Fig. 2, the literature's standard measures and ι^{phy} produce statistically insignificant q-slopes and R^2 values near zero. In stark contrast, intangible and total investment both have highly significant q-slopes, and they deliver R^2 values of 57% and 61%, respectively. These R^2 values are even higher than the 46% R^2 that

[13] To be precise, we use the Hall (2001) data on q^* and ι^* and the Corrado and Hulten (2014) data on $A = K^{phy}/(K^{phy} + K^{int})$ and $B = I^{phy}/(I^{phy} + I^{int})$. We compute $q^{tot} = q^* A$, $\iota^{phy} = \iota^* A$, $\iota^{tot} = \iota^{phy}/B$, and $\iota^{int} = \iota^{tot} - \iota^{phy}$.

Figure 8: Investment-q relation in macro data.



This figure plots Tobin's q (solid lines) and the investment rates (dashed lines) over time for the aggregate US economy. Panel A uses data from Hall (2001) and shows standard measures that exclude intangible capital. Standard q (q^*) is aggregate market value scaled by the physical capital stock. Standard investment (ι^*) equals physical investment scaled by the physical capital stock. Panels B–D also use data from Corrado and Hulten (2014). Total q is aggregate market value scaled by total capital, the sum of the physical and intangible capital stocks. Panel B shows ι^{phy}, physical investment scaled by total capital. Panel C shows ι^{int}, intangible investment scaled by total capital. Panel D shows $\iota^{tot} = \iota^{phy} + \iota^{int}$. For each graph, the left axis is the value of q and the right axis is the investment rate.

Philippon (2009) obtains by regressing the standard investment measure on bond q (Panel B). Judging by R^2, the classic q theory fits the data much better when we include intangible capital, because we are better able to explain the low-frequency trends in q and investment. Put differently, the literature's standard investment measure suffers from a low-frequency error—the omission of intangibles—that trends strongly with q over time.

How well can q explain higher-frequency variation in investment? Panels C and D answer this question by rerunning the previous regressions in four-quarter differences. As in our firm-level analysis, total q now explains physical and intangible investment roughly equally well, and it explains total investment even better. As before, intangible investment has a lower q-slope than physical investment, indicating higher convex adjustment costs. Bond q, though, is much better than total q at explaining changes in investment. In differences, bond q also explains physical investment better than intangible investment. Philippon (2009) offers one potential explanation: Growth options affect stocks more than bonds, and growth options affect intangible investment more than physical investment. Put differently, physical and intangible capital can have different values of marginal q. Bond q could be a better proxy for physical capital's marginal q, whereas the traditional q measures, which use stock prices, could be better proxies for intangible capital's marginal q. A second possible explanation is about sample selection. Firms with more intangible investment typically hold less debt, so they contribute less to the aggregate bond q measure.

To summarize, at the macro level, including intangibles makes q explain the level of investment much better, meaning the classic q theory fits the data much better than previously believed. When we try to explain changes in investment, the macro results look more like our firm-level results. Bond q is still better at explaining physical investment as well as changes in investment.

Table 17: Time series macro regressions

Results are from 141 quarterly observations from aggregate US data, from 1972Q2 to 2007Q2. Each column uses a different investment measure. Standard q (q^*) equals the lagged aggregate stock and bond market value divided by the physical capital stock. Hall (2001) computes these measures from the flow of funds. Total q includes intangible capital by multiplying physical q by the ratio of physical to total capital. The ratio is from the Corrado and Hulten (2014) aggregate US data. Bond q is constructed by applying the structural model of Philippon (2009) to bond maturity and yield data. These data are from Philippon's website. Newey-West standard errors with autocorrelation up to 12 quarters are in parentheses. Standard errors for the ordinary least squares (OLS) R^2 values are computed via bootstrap.

		Investment scaled by total capital (K^{tot})		
	CAPX / PPE (ι^*)	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})
Panel A: Regressions in levels				
Total q (q^{tot})		-0.001	0.019	0.017
		(0.003)	(0.003)	(0.003)
Standard q (q^*)	0.002			
	(0.003)			
OLS R^2	0.035	0.014	0.570	0.610
	(0.034)	(0.030)	(0.026)	(0.040)
Panel B: Regressions in levels with bond q				
Bond q	0.061	0.049	0.006	0.055
	(0.009)	(0.011)	(0.039)	(0.032)
OLS R^2	0.462	0.347	0.001	0.139
	(0.059)	(0.050)	(0.013)	(0.060)
Panel C: Regressions in four-quarter differences				
Total q (q^{tot})		0.007	0.004	0.01
		(0.002)	(0.001)	(0.003)
Standard q (q^*)	0.007			
	(0.002)			
OLS R^2	0.124	0.106	0.096	0.121
	(0.057)	(0.052)	(0.056)	(0.060)
Panel D: Regressions in four-quarter differences with bond q				
Bond q	0.056	0.043	0.017	0.060
	(0.006)	(0.005)	(0.004)	(0.008)
OLS R^2	0.606	0.620	0.235	0.530
	(0.053)	(0.059)	(0.074)	(0.070)

2.7. Robustness

This section describes our results' robustness across different subsamples, empirical measures, and estimators. We also explain why our main results are not mechanical.

2.7.1. Robustness of main results across subsamples

Tables 14–16 show that our main results are quite robust across subsamples. Compared with physical investment, intangible investment has a lower q-slope in all ten subsamples (Panel A). We always see larger q-slopes for physical investment in the specification with ι^{phy} compared with the specification with CAPX/PP&E (Panel A). Total q always explains total investment better than it explains either physical or intangible investment and better than standard q explains standard investment (Panel B). This result means that including intangibles produces a better proxy for investment opportunities even in subsamples with less intangible capital, such as the manufacturing industry. In the full sample (Table 13), total q explains intangible investment slightly better than physical investment. We see the reverse in four of ten subsamples, so we conclude that total q explains physical and intangible investment roughly equally well.

The improvement in model fit from including intangible capital is especially large in subsamples with more intangible capital, which is a useful consistency check. Consider the increase in R^2 when moving from the regression that ignores intangibles (Specification 4 in the tables) to the regression that uses ι^{tot} and q^{tot} (Specification 3). In Table 14, the increase in R^2 is 0.174 (58%) in the highest intangible quartile, but just 0.050 (27%) in the lowest quartile. This pattern is mainly driven by τ^2, which increases by 0.284 (65%) in the top quartile but decreases by a statistically insignificant 0.018 (3%) in the lowest quartile. This result means that total q is a better proxy for true q, especially in firms with the most intangible capital. These same patterns are also present, but less dramatic, across industry and year subsamples.

Table 18: Robustness: what fraction of SG&A expense is an investment?

Results are from regressions of three investment measures on lagged total q and firm fixed effects. Slopes on total q are from the cumulant estimator. Within-firm R^2 is from the ordinary least squares (OLS) estimator that also includes year fixed effects. The selling, general and administrative (SG&A) multiplier is the fraction of SG&A assumed to represent an investment. Our main analysis uses a 0.3 multiplier. For each multiplier value, we reestimate the intangible investment and capital stocks in the data. Because physical investment, total investment, and total q are scaled by total capital, their values also depend on the SG&A multiplier. Each regression uses 141,800 firm-year Compustat observations from 1975 to 2011. * indicates value used in main analysis

	Investment scaled by total capital (K^{tot})					
SG&A	Physical (ι^{phy})		Intangible (ι^{int})		Total (ι^{tot})	
Multiplier	q-slope	OLS R^2	q-slope	OLS R^2	q-slope	OLS R^2
0.0	0.060	0.223	0.021	0.147	0.064	0.277
0.1	0.064	0.217	0.025	0.256	0.074	0.307
0.2	0.067	0.213	0.032	0.276	0.081	0.320
0.3*	0.070	0.209	0.037	0.279	0.086	0.327
0.4	0.072	0.206	0.043	0.278	0.092	0.331
0.5	0.075	0.203	0.048	0.274	0.097	0.333
0.6	0.077	0.201	0.054	0.270	0.103	0.335
0.7	0.078	0.200	0.06	0.266	0.108	0.335
0.8	0.080	0.198	0.065	0.262	0.113	0.334
0.9	0.082	0.197	0.071	0.257	0.118	0.333
1.0	0.084	0.196	0.075	0.253	0.122	0.332

2.7.2. What fraction of SG&A is an investment?

Arguably, the strongest assumptions in our intangible-capital measure are that $\lambda = 30\%$ of SG&A represents an investment and λ is constant across firms and time. Table 18 shows that our main conclusions go though, at least qualitatively, when we use different values of λ ranging from zero to 100%. When λ is zero, firms' intangible capital comes exclusively from R&D. No matter what λ value we assume, we find larger q-slopes for physical investment, roughly equal R^2 for physical and intangible investment, and the highest R^2 for total investment. Intangible investment has its largest R^2 when $\lambda = 30\%$, meaning the data seem to prefer the λ value we use in our main analysis. The R^2 is considerably lower (15% versus 28%) if $\lambda = 0$, so the data do prefer counting at least part of SG&A as investment.

Instead of assuming $\lambda = 30\%$, we can let the data reveal λ's value. The structural parameter λ affects both the investment and q measures. We estimate λ along with the q-slope and firm fixed effects by maximum likelihood, applied to the ι^{tot} regression. The estimated λ values are 0.38 in the consumer industry, 0.51 in the high-tech industry, and 0.24 in the health care industry, which are all in the neighborhood of our assumed 0.3 value. However, we do not push these λ estimates strongly, for three reasons. First, the investment-q relation is not the ideal setting for identifying λ. Second, the estimation imposes two very strong identifying assumptions: The linear investment-q model is true, and we measure all variables perfectly. Finally, the λ estimate in the manufacturing industry is constrained at 1.0, which is implausibly large and likely a symptom of the previous two issues. The main message from this subsection, though, is that our main conclusions hold regardless of the λ value we use.

2.7.3. Alternate measures of intangible capital

In addition to varying the SG&A multiplier λ, we try nine other variations on our intangible-capital measure. We vary $\delta_{SG\&A}$, the depreciation rate for organization capital; we vary $\delta_{R\&D}$, the depreciation rate for knowledge capital; we exclude Goodwill from firms' intangible capital; we exclude all balance sheet intangibles, which brings us closer to existing measures from the literature; we set firms' starting intangible capital stock to zero; and we estimate firms' starting intangible capital stock using a perpetuity formula, like Falato, Kadyrzhanov, and Sim (2013). We also drop the first five years of data for each firm, which makes the choice of starting intangible capital stock less important. We also try dropping the 47% of firm-years with missing R&D from our regressions. Table 19 provides details about these variations and their results. Although magnitudes vary somewhat, our main results still hold in all these variations: Total q explains physical and intangible investment roughly equally well, total q explains total investment even better, and intangible investment always has a lower q-slope.

Table 19: Robustness: Robustness: alternate measures of intangible capital

Results are from regressions of our three main investment measures on lagged total q and firm fixed effects. The column labels indicate the investment measure used. Slopes on q are from the cumulant estimator. We report the within-firm R^2 from the ordinary least squares (OLS) estimator that also includes year fixed effects. The first row reproduces results from Table 13 with our main intangible-capital measure. Rows 2–10 show results using alternate measures of intangible capital. Rows 2 and 3 use alternate values of $\delta_{SG\&A}$, the depreciation rate for organization capital. Rows 4–6 use alternate values of $\delta_{R\&D}$, the depreciation rate for knowledge capital. Row 7 excludes goodwill from balance sheet intangibles. Row 8 excludes all balance sheet intangibles. Row 9 assumes firms have no intangible capital before entering Compustat, which corresponds to setting $G_{i0} = 0$ in Eq. (2.11). Row 10 estimates firms' starting intangible capital using a perpetuity formula that assumes the firm has been alive forever before entering Compustat, as in Falato, Kadyrzhanova and Sim (FKS, 2013). The initial stock of knowledge capital (for example) is $G_{i0} = R\&D_{i1}/\delta_{R\&D}$, where $R\&D_{i1}$ is the research and development (R&D) amount in firm i's first Compustat record. Row 11 uses our main intangible-capital measures but drops each firm's first five years of data. Row 12 uses use our main intangible-capital measure but drops firm-year observations with missing R&D. Data are from Compustat from 1975 to 2011.

	Investment scaled by total capital (K^{tot})						
	Physical (ι^{phy})		Intangible (ι^{int})		Total (ι^{tot})		
Specification	q-slope	OLS R^2	q-slope	OLS R^2	q-slope	OLS R^2	N
1. Main results (Table 3)	0.070	0.209	0.037	0.279	0.086	0.327	141,800
2. $\delta_{SG\&A} = 10\%$	0.071	0.213	0.038	0.294	0.088	0.337	141,800
3. $\delta_{SG\&A} = 30\%$	0.069	0.207	0.037	0.270	0.086	0.322	141,800
4. $\delta_{R\&D} = 10\%$	0.073	0.209	0.038	0.285	0.089	0.329	141,800
5. $\delta_{R\&D} = 15\%$	0.072	0.209	0.038	0.282	0.088	0.328	141,800
6. $\delta_{R\&D} = 20\%$	0.070	0.208	0.038	0.279	0.087	0.327	141,800
7. Exclude Goodwill	0.070	0.209	0.037	0.282	0.086	0.329	141,800
8. Exclude balance sheet intangibles	0.063	0.199	0.035	0.248	0.079	0.306	141,800
9. Zero initial intangible capital	0.069	0.217	0.040	0.297	0.089	0.343	141,800
10. FKS initial multiplier	0.073	0.193	0.033	0.238	0.084	0.293	141,800
11. Drop first five years per firm	0.069	0.132	0.037	0.153	0.078	0.210	82,174
12. Exclude observations with missing R&D	0.062	0.267	0.036	0.359	0.081	0.411	75,426

2.7.4. *Alternate estimators*

In addition to using the cumulant estimator to obtain unbiased q-slopes, we use the Biorn (2000) and Arellano and Bond (1991) instrumental variable (IV) estimators. Both estimators take first differences of the linear investment-q model and then use lagged regressors as instruments for the q proxy. Erickson and Whited (2012) show that these IV estimators are biased if measurement error is serially correlated, which is likely in our setting. This bias is probably most severe in the standard regressions that omit intangible capital, as omitting intangible capital is an important source of measurement error, and a firm's intangible capital stock is highly serially correlated. Because the cumulant estimators are robust to serially correlated measurement error, we prefer them over the IV estimators. The IV estimators generate similar conclusions about adjustment costs. They produce lower q-slopes for ι^{int} than ι^{phy} and lower q-slopes for ι^{*} than ι^{phy} (Online Appendix).

2.7.5. *A mechanical result?*

Is it mechanical that total q explains total investment better than standard q explains standard investment? A potential concern is that moving from the latter regression to the former requires multiplying both sides of the regression by K^{phy}/K^{tot}. Multiplying both sides of a regression by the same variable can, but does not necessarily, increase the R^2 even if that variable is pure noise.

Our result is not mechanical or obvious, however. Multiplying both sides of the literature's standard regression by K^{phy}/K^{tot} produces the regression of ι^{phy} on q^{tot}, shown in Column 1 of Table 13. Contrary to the concern, that regression gets a slightly *lower* R^2, τ^2, and ρ^2 value than the standard regression (last column in Table 13). Moving to the regression of ι^{tot} on q^{tot} requires further multiplying ι^{phy}, but not q^{tot}, by the ratio of total to physical investment. This change would further reduce the R^2 if intangible investment were noise, but instead R^2 increases. Moreover, if our measure of intangible investment were just noise, we would not find that it is well explained by q and co-moves with physical investment.

The Online Appendix presents a placebo simulation analysis showing that our main results would not obtain if our intangible capital measures were pure noise with similar statistical properties.

2.8. Conclusion

The neoclassical theory of investment has been applied almost exclusively to physical capital. We show that the theory is also relevant for intangible capital, which increasingly dominates the US economy. In both our theory and firm-level data, physical and intangible investment co-move strongly, and they are explained roughly equally well by Tobin's q. Compared with physical capital, intangible capital's convex adjustment costs are roughly twice as large, meaning intangible capital responds more slowly to changes in investment opportunities. In macro data, Tobin's q explains the level of intangible investment many times better than physical investment. The neoclassical theory performs significantly better in firms, industries, and years with more intangible capital.

Tobin's q is "arguably the most common regressor in corporate finance" (Erickson and Whited, 2012, p. 1286). Guided by our theory, we provide a new Tobin's q measure that accounts for intangible capital, and we show that it is a superior proxy for both physical and intangible investment opportunities. This new Tobin's q measure offers a simple way to improve corporate finance regressions without additional econometrics. A benefit of the new measure is that it can be easily computed for the full Compustat sample. Data on our Tobin's q measure and firms' intangible capital can be downloaded from WRDS.

This paper revisits the basic facts about investment, Tobin's q, and cash flow while accounting for intangible capital. We believe this is an important step, because a vast investment literature in corporate finance and macroeconomics is built upon these facts. Important next steps include understanding how physical and intangible capital interact, how they face different prices for different firms in different periods, how they respond differently to growth options and financial constraints, and how they show up differently in firms' market values. Why the classic q-theory fits the data better in high-intangible settings is also an interesting open question. Finally, there is more work to do on measuring intangible capital.

Appendix A. Proofs

A.1. Proof of Prediction 1

Dropping firm subscripts, we can write the value function as

$$V_t = \max_{\{I_{t+s}^{phy}, I_{t+s}^{int}\}} \int_0^\infty E_t\{ \quad K_{t+s}^{tot}[H(\varepsilon_{t+s}) - \frac{\gamma^{phy}}{2}\left(\frac{I_{t+s}^{phy}}{K_{t+s}^{tot}}\right)^2 - \frac{\gamma^{int}}{2}\left(\frac{I_{t+s}^{int}}{K_{t+s}^{tot}}\right)^2$$

$$- \left(p_{t+s}^{phy} + \zeta^{phy}\right)\frac{I_{t+s}^{phy}}{K_{t+s}^{tot}} - \left(p_{t+s}^{int} + \zeta^{int}\right)\frac{I_{t+s}^{int}}{K_{t+s}^{tot}}]\}. \tag{2.18}$$

Total capital follows

$$dK_t^{tot} = K_t^{tot}\left(\frac{I_t^{phy}}{K_t^{tot}} + \frac{I_t^{int}}{K_t^{tot}} - \delta\right)dt. \tag{2.19}$$

Following the same argument as in Appendix A of Abel and Eberly (1994), firm value must be proportional to total capital K^{tot} :

$$V\left(K^{phy}, K^{int}, \varepsilon, p^{phy}, p^{int}\right) = K^{tot} q^{tot}\left(\varepsilon, p^{phy}, p^{int}\right). \tag{2.20}$$

Differentiating this equation with respect to K^{phy} and K^{int} yields Eq. (2.5).

A.2. Proof of Prediction 2

Following a similar proof as in Abel and Eberly (1994), one can derive the Bellman equation and take first-order conditions with respect to each investment rate to obtain

$$q_t^{tot} = \frac{\partial}{\partial I_t^m} c^m\left(I_t^m, K_t^{tot}, p_t^m\right) = p_t^m + \zeta^m + \gamma^m \frac{I_t^m}{K^{tot}}, \quad m = phy,\ int. \tag{2.21}$$

Rearranging yields Eqs. (2.6) and (2.7).

A.3. Proof of Prediction 4

Multiplying both sides of Eq. (2.6) by $K_{it}^{tot}/K_{it}^{phy}$ yields

$$\iota_{it}^* = \frac{I_{it}^{phy}}{K_{it}^{phy}} = \frac{1}{\gamma^{phy}}\left(q_{it}^* - \frac{K_{it}^{tot}}{K_{it}^{phy}}\left(\zeta_i^{phy} + p_{it}^{phy}\right)\right). \tag{2.22}$$

Now consider a regression of ι_{it}^{*} on q_{it}^{*} and firm and time FEs. The residual in that regression, ε_{it}^{*}, equals the residual from a regression of $-\frac{1}{\gamma^{phy}}\frac{K_{it}^{tot}}{K_{it}^{phy}}\left(\zeta_{i}^{phy}+p_{it}^{phy}\right)$ on firm and time FEs. This residual is nonzero and, hence, the regression's R^2 is less than 100%, because the ratio $K_{it}^{tot}/K_{it}^{phy}$ cannot be fully explained by firm and time fixed effects. To see this last claim, define $\omega_{it}=K_{it}^{tot}/K_{it}^{phy}$. By Ito's lemma, ω evolves according to

$$\frac{d\omega_{it}}{\omega_{it}}=\left[\iota_{it}^{phy}\left(1-\omega_{it}\right)+\iota_{it}^{int}\right]dt. \tag{2.23}$$

The evolution of ω_{it} cannot be fully be explained by firm and time FEs, because it depends on the investment rates ι_{it}^{phy} and ι_{it}^{int}, which depend on q_{it}^{tot} and, hence, ε_{it}, which cannot be fully explained by the FEs. Furthermore, the error term ε_{it}^{*} is negatively correlated to the regressor $q_{it}^{*}=q_{it}^{tot}K_{it}^{tot}/K_{it}^{phy}$, because $K_{it}^{tot}/K_{it}^{phy}$ multiplies both terms, albeit with a negative sign in ε_{it}^{*}. Because the error term is negatively related to the regressor, the regression produces downward-biased estimates of $1/\gamma^{phy}$.

A.4. Proof of last prediction

Set $d\omega_{it}=0$ in Eq. (2.23) and solve for the equilibrium value, $\overline{\omega}$:

$$\overline{\omega}=\frac{\iota_{it}^{int}+\iota_{it}^{phy}}{\iota_{it}^{phy}}=\frac{\frac{1}{\gamma^{int}}\left(q_{it}^{tot}-\zeta-p_{it}\right)+\frac{1}{\gamma^{phy}}\left(q_{it}^{tot}-\zeta-p_{it}\right)}{\frac{1}{\gamma^{phy}}\left(q_{it}^{tot}-\zeta-p_{it}\right)}=\frac{\gamma^{phy}+\gamma^{int}}{\gamma^{int}}. \tag{2.24}$$

The last prediction follows, because $K^{int}/K^{tot}=1-1/\omega$.

Appendix B. Measuring intangible capital

B.1. Measuring SG&A

We measure SG&A as Compustat variable *xsga* minus *xrd* minus *rdip*. We add the following screen: When *xrd* exceeds *xsga* but is less than *cogs*, or when *xsga* is missing, we measure SG&A as *xsga* with no further adjustments or zero if *xsga* is missing.

The logic behind this formula is as follows. According to the Compustat manual, *xsga* includes R&D expense unless the company allocates R&D expense to cost of goods sold (COGS). For example, *xsga* often equals the sum of Selling, General and Administrative and Research and Development on the Statement of Operations from firms' 10-K filings. To isolate (non-R&D) SG&A, we must subtract R&D from *xsga* when Compustat adds R&D to *xsga*. There is a catch: When a firm externally purchases R&D on products not yet being sold, this R&D is expensed as In-Process R&D and does not appear on the balance sheet. Compustat adds to *xsga* only the part of R&D not representing acquired In-Process R&D, so our formula subtracts *rdip* (In-Process R&D Expense), which Compustat codes as negative. We find that Compustat almost always adds R&D to *xsga*, which motivates our formula above. Standard & Poor's explained in private communication that, "in most cases, when there is a separately reported *xrd*, this is included in *xsga*." As a further check, we compare the Compustat records and 10-K filings for a random sample of one hundred firm-year observations with non-missing *xrd*. We find that Compustat includes R&D in *xsga* in 90 out of one hundred cases, partially includes it in *xsga* in one case, and includes it in COGS in seven cases. Two cases remain unclear even after asking the Compustat support team. The screen above lets us identify obvious cases in which *xrd* is part of COGS. This screen catches six of the seven cases in which *xrd* is part of COGS. Unfortunately, identifying the remaining cases is impossible without reading SEC filings. We thank the Compustat support team from Standard & Poors for their help in this exercise.

We set *xsga*, *xrd*, and *rdip* to zero when missing. For R&D and SG&A, we make exceptions in years when the firm's assets are also missing. For these years, we interpolate these two variables using their nearest non-missing values. We use these interpolated values to compute capital stocks but not regressions' dependent variables.

B.2. Measuring firms' initial capital stock

This section explains how we estimate the stock of knowledge and organization capital in firm i's first non-missing Compustat record. We describe the steps for estimating the initial knowledge-capital stock. The method for organization capital is similar. Broadly, we estimate firm i's R&D spending in each year of life between the firm's founding and its first non-missing Compustat record, denoted year one. Our main assumption is that the firm's pre-IPO R&D grows at the average rate across pre-IPO Compustat records. We then apply the perpetual inventory method to these estimated R&D values to obtain the initial stock of knowledge capital at the end of year zero. The seven steps are as follows.

1. Define age since IPO as number of years elapsed since a firm's IPO. Using the full Compustat database, compute the average log change in R&D in each yearly category of age since IPO. Apply these age-specific growth rates to fill in missing R&D observations before 1977.

2. Using the full Compustat database, isolate records for firms' IPO years and the previous two years. (Not all firms have pre-IPO data in Compustat.) Compute the average log change in R&D within this pre-IPO subsample, which equals 0.348. (The corresponding pre-IPO average log change in SG&A equals 0.333.)

3. If firm i's IPO year is in Compustat, go to Step 5. Otherwise go to the next step.

4. This step applies almost exclusively to firms with IPOs before 1950. Estimate firm i's R&D spending in each year between the firm's IPO year and first Compustat year, assuming the firm's R&D grows at the average age-specific rates estimated in Step 1.

5. Obtain data on firm i's founding year from Jay Ritter's website. For firms with missing founding year, estimate the founding year as the minimum of (a) the year of the firm's first Compustat record and (b) firm's IPO year minus eight, which is the median age between founding and IPO for IPOs from 1980 to 2012 (from Jay Ritter's website).

6. Estimate the firm i's R&D spending in each year between the firm's founding year and IPO year assuming the firm's R&D grows at the estimated pre-IPO average rate from Step 2.

7. Assume the firm is founded with no capital. Apply the perpetual inventory method in Eq. (2.11) to the estimated R&D spending from the previous steps to obtain G_{i0}, the stock of knowledge capital at the beginning of the firm's first Compustat record.

We only use estimated R&D and SG&A values to compute firms' initial stocks of intangible capital. We never use estimated R&D in a regression's dependent variable.

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